Vector Autoregressions (VARs)

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Outline

1. Literature
2. What are VARs used for?
3. Description of basic VAR models
4. Impulse response functions
5. Forecasting
6. Forecast error variance decomposition (FEVD)
7. Historical decomposition (HD)

Literature

- Seminal paper: Sims [1980] $\longrightarrow$ Nobel-prize, 2011
- Estimation of VARs: Canova [2007]
- Structural VARs: Stock and Watson [2001] and Rubio-Ramirez et al. [2010]
- Bayesian VARs: Blake and Mumtaz [2012]

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  4. Data description: does unemployment help predict inflation better than the interest rate?

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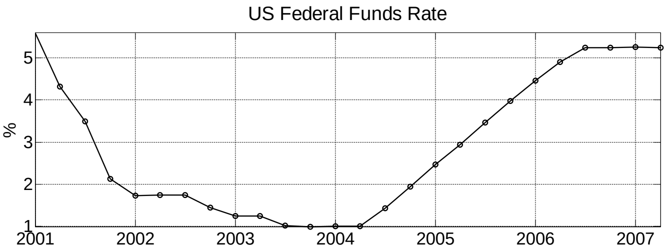
- A popular narrative is that the low interest rate environment in the US post 9/11 contributed to the balance sheet expansion of the financial sector (housing bubble)

Figure : Pre-crisis US Monetary Policy



- First, it is unclear whether the low interest rates were a natural consequence of the Fed behaving optimally by responding to deteriorating business cycle conditions via cutting rates.
  - Could low rates be explained by the systematic part of policy, or were low rates in fact the result of non-systematic policy shocks?
  - Identifying monetary policy shocks and separating them from the systematic parts of policy is needed to understand historical interest rate dynamics.

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  - Could low rates be explained by the systematic part of policy, or were low rates in fact the result of non-systematic policy shocks?
  - Identifying monetary policy shocks and separating them from the systematic parts of policy is needed to understand historical interest rate dynamics.
- Second, it is unclear what the partial impact of monetary policy shocks has been on balance sheet expansions, given other factors (e.g. structural shocks related to increased housing demand and commodity price booms)
  - One must disentangle monetary policy shocks from other structural shocks in order to assess the relative importance of monetary policy

What is a VAR

- Suppose we have data on C (consumption) and Y (output). The VAR model of order 1 (VAR(1)) is written as:

$$
\begin{aligned}
C_t =& \lambda + \beta_1 C_{t-1} + \beta_2 Y_{t-1} + e_t \\
Y_t =& \alpha + \rho_1 C_{t-1} + \rho_2 Y_{t-1} + \nu_t \\
\Sigma =& E\left(e_t, \nu_t\right)
\end{aligned}
\tag{1.1}
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- Each variable depends on the lags of the two $\longrightarrow$ each variable is endogenous

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- Just N linear regression
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$$Y = \begin{pmatrix} C_2 & Y_2 \\ C_3 & Y_3 \\ \vdots & \vdots \\ C_T & Y_T \end{pmatrix}, \qquad X = \begin{pmatrix} 1 & C_1 & Y_1 \\ 1 & C_2 & Y_2 \\ 1 & \vdots & \vdots \\ 1 & C_{T-1} & Y_{T-1} \end{pmatrix}$$

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- The residuals are used to estimate the variance-covariance matrix:

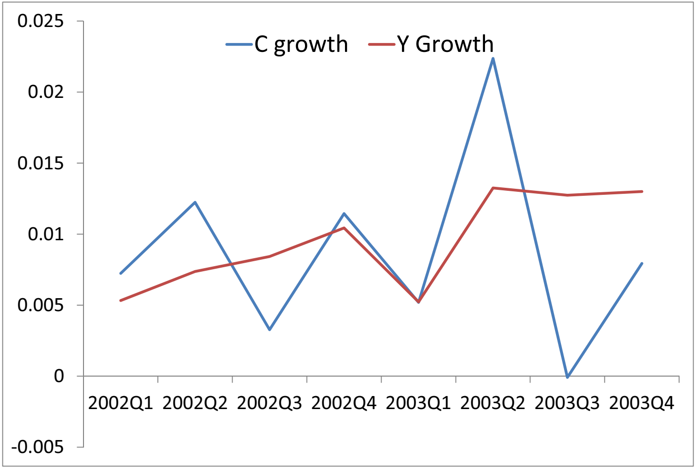
$$\Omega = Y - X\hat{B} \tag{1.3}$$

$$\Sigma = \frac{\Omega'\Omega}{T - K}$$

Estimating a VAR model

Illustration

Figure : Growth of real consumption and output in the UK, 2002Q1-2003Q4



- The data:

$$UK\ \ Data = \begin{bmatrix} Period & C\ \ growth & Y\ \ growth \\ 2002Q1 & 0.0072 & 0.0053 \\ 2002Q2 & 0.0122 & 0.0073 \\ 2002Q3 & 0.0032 & 0.0084 \\ 2002Q4 & 0.0114 & 0.0104 \\ 2003Q1 & 0.0051 & 0.0052 \\ 2003Q2 & 0.0223 & 0.0132 \\ 2003Q3 & -8.8E-05 & 0.0127 \\ 2003Q4 & 0.0079 & 0.0129 \end{bmatrix} \tag{1.4}$$

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- Y and X matrices:

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$$\hat{B} = \begin{bmatrix} 0.024 & 0.008 \\ -0.46 & -0.09 \\ 1.254 & 0.309 \end{bmatrix} \tag{1.7}$$

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- Using the data matrices covering 7 quarters:

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- Plug the estimates back into our VAR model 1.1:

$$\begin{aligned} C_t &= 0.024 - 0.46 C_{t-1} + 1.254 Y_{t-1} + e_t \\ Y_t &= 0.008 - 0.09 C_{t-1} + 0.309 Y_{t-1} + \nu_t \end{aligned} \tag{1.8}$$

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- Degrees of freedom $\longrightarrow$ a lot of coefficients can lead to imprecise estimates

Lag length in a VAR

- Total number of parameters, K, in a VAR with
  - N number of endogenous variables, and
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$$K = ((N \times P) + 1) \times N \tag{1.9}$$

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- If time series is short, then the choice of P is crucial:

$$\begin{aligned} AIC &= \ln det(\Sigma) + \frac{2N^2 P}{T} \\ SIC &= \ln det(\Sigma) + \frac{N^2 P \ln T}{T} \end{aligned} \tag{1.10}$$

- Choose smallest AIC/SIC after estimating from $P = 1 \ldots p$

Inference VARs: Impulse Response Functions

- Consider our VAR(1) model:

$$\begin{aligned} C_t &= \lambda + \beta_1 C_{t-1} + \beta_2 Y_{t-1} + e_t \\ Y_t &= \alpha + \rho_1 C_{t-1} + \rho_2 Y_{t-1} + \nu_t \end{aligned} \tag{2.1}$$

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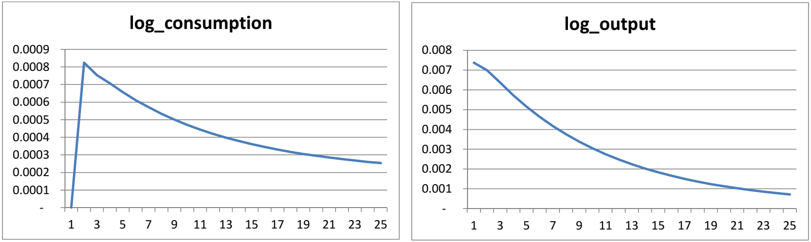
- Assume the coefficients are known: What is the impact on C if there is a shock to Y?
- The impact of this is referred to as the impulse response function (IRF)

Figure : Example for IRFs



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- Consider the following experiment:
  - Time 0: both y and x are zero and $c = 0$
  - Time 1: one unit increase in x

Impulse Response Functions

Tracing out the impact

- Initial Shock:

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- Period 2:

$$\begin{pmatrix} y_2 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} y_1 \\ x_1 \end{pmatrix} = \begin{pmatrix} b_{11}y_1 + & b_{12}x_1 \\ b_{21}y_1 + & b_{22}x_1 \end{pmatrix} = \begin{pmatrix} b_{12} \\ b_{22} \end{pmatrix} \tag{2.5}$$

Impulse Response Functions

- Period 3:

$$\begin{pmatrix} y_3 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} y_2 \\ x_2 \end{pmatrix} \tag{2.6}$$

$$\begin{pmatrix} y_3 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}^2 \begin{pmatrix} y_1 \\ x_1 \end{pmatrix} \tag{2.7}$$

$$\begin{pmatrix} y_3 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_{11}^2 + b_{12}b_{21} & b_{11}b_{12} + b_{12}b_{22} \\ b_{21}b_{11} + b_{22}b_{21} & b_{21}b_{12} + b_{22}^2 \end{pmatrix} \begin{pmatrix} y_1 \\ x_1 \end{pmatrix} \tag{2.8}$$

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- The impulse response for period i is:

$$\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}^{i-1} \tag{2.10}$$

Forecasting

- VARs are useful for forecasting as all variables are endogenous:

$$
\begin{pmatrix} \hat{Y}_{T+1} \\ \hat{C}_{T+1} \end{pmatrix} = \begin{pmatrix} \hat{b}_{11} & \hat{b}_{12} \\ \hat{b}_{21} & \hat{b}_{22} \end{pmatrix} \begin{pmatrix} Y_T \\ C_T \end{pmatrix}
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- Useful as a short term forecasting tool $\longrightarrow$ but little theoretical underpinning

Forecasting

- Conditional forecasting is a useful application
- Assume some of the variables in the VAR are exogenous in the future (e.g. pegged interest rate)
- These may be variables for which data is more readily available or a future path of these variables is available

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- In conditional forecasts we calculate future shocks that give us the constrained forecast (e.g. Compass - BoE forecasting platform)

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- Imagine you make a forecast for the variables in the VAR
- Because of shocks, there will be an error in forecasting
- FEVD refers to working out how much each shock contributed to the forecast error variance

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- The variables are going to be hit by the two shocks in our model
- The IRF and the scale of the shocks will determine how much each variable moves

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Table : Shock contributions

Shock	e_t	ν_t
Movement in period 1	$\sigma_e \Psi_{e,1}$	$\sigma_\nu \Psi_{\nu,1}$
Contribution to variance	$(\sigma_e \Psi_{e,1})^2$	$(\sigma_\nu \Psi_{\nu,1})^2$

Forecast Error Variance Decomposition (FEVD)

- 2 periods in the future – impact of the shocks in period 1 and period 2 and so on

Table : Shock contributions

Shock	e_t	ν_t
Movement in period 2	$\sigma_e \Psi_{e,1} + \sigma_e \Psi_{e,2}$	$\sigma_\nu \Psi_{\nu,1} + \sigma_\nu \Psi_{\nu,2}$
Contribution to variance	$(\sigma_e \Psi_{e,1})^2 + (\sigma_e \Psi_{e,2})^2$	$(\sigma_\nu \Psi_{\nu,1})^2 + (\sigma_\nu \Psi_{\nu,2})^2$

Historical Decomposition

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} \nu_{1,t} \\ \nu_{2,t} \end{pmatrix} \tag{5.1}$$

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Historical Decomposition

- Historical decomposition measures the contribution of each shock to the level of each series:

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \overbrace{\sum_{i=0}^{T-1} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}^i \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}^T \begin{pmatrix} y_0 \\ x_0 \end{pmatrix}}^{Base\ or\ trend} \\ + \underbrace{\sum_{i=0}^{T-1} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}^i \begin{pmatrix} \nu_{1,t-i} \\ \nu_{2,t-i} \end{pmatrix}}_{Contributions\ of\ the\ 2\ shocks} \tag{5.4}$$

Historical Decomposition

- Historical decomposition presented as bar chart where the bars represent contribution by the shocks and the trend

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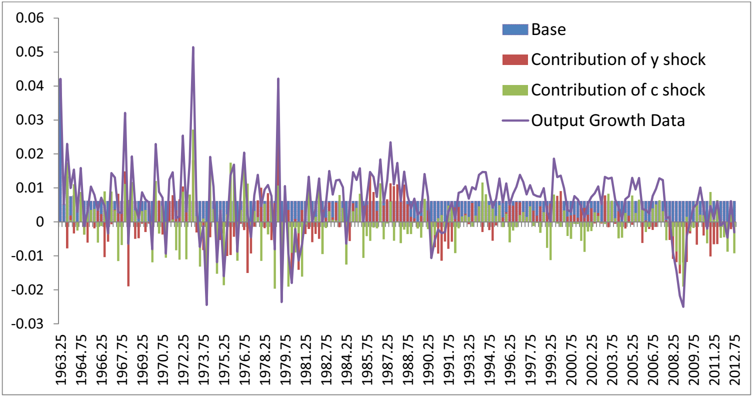
Figure : Example for Historical Decomposition



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- Identification is needed to "make structural shocks out of" reduced-form residuals
- Structural VARs make various identification assumptions to achieve meaningful decompositions

References

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