Structural Vector Autoregressions (SVARs)

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Literature

- Structural VARs: Stock and Watson [2001] and Rubio-Ramirez et al. [2010]
- Monetary policy shocks: Christiano et al. [1999]
- Sign restrictions: Uhlig [2005] and Fry and Pagan [2011]
- Narrative measures: Romer and Romer [2004] and Coibion [2012]
- Time-varying parameters and stochastic volatility: Primiceri [2005] and Mumtaz and Plassmann [2013]

Structural Vector Autoregressions

- IRFs, FEVDs and Historical decompositions are only interesting if:
  - Shocks in the VAR are uncorrelated
  - Shocks in the VAR have economic interpretation
- This not straightforward $\longrightarrow$ SVARs are a way of ensuring that these conditions hold

Consider the following (highly stylised) 'structural' model:

$$\begin{aligned}
Supply\ \ curve: \pi_t &= \beta\pi_{t-1} + \lambda y_t + \overbrace{e_t}^{Supply\ \ shock} \\
Demand\ \ curve: y_t &= \theta y_{t-1} + bR_t + \overbrace{\nu_t}^{Demand\ \ shock} \\
MP\ \ rule: R_t &= \kappa\pi_t + \omega y_t + \rho R_{t-1} + \overbrace{\varepsilon_t}^{Policy\ \ shock}
\end{aligned} \tag{1.1}$$

Rearrange the model by putting all contemporaneous variables on the left-hand-side:

$$\begin{aligned}
\pi_t - \lambda y_t &= \beta \pi_{t-1} + \overbrace{e_t}^{Supply\ \ shock} \\
y_t - bR_t &= \theta y_{t-1} + \overbrace{\nu_t}^{Demand\ \ shock} \\
R_t - \kappa \pi_t - \omega y_t &= \rho R_{t-1} + \overbrace{\varepsilon_t}^{Policy\ \ shock}
\end{aligned} \tag{1.2}$$

Write out:

$$\underbrace{\begin{pmatrix} 1 & -\lambda & 0 \\ 0 & 1 & -b \\ -\kappa & -\omega & 1 \end{pmatrix}}_{\phi_0} \begin{pmatrix} \pi_t \\ y_t \\ R_t \end{pmatrix} = \underbrace{\begin{pmatrix} \beta & 0 & 0 \\ 0 & \theta & 0 \\ 0 & 0 & \rho \end{pmatrix}}_{A_1} \begin{pmatrix} \pi_{t-1} \\ y_{t-1} \\ R_{t-1} \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_t \\ \nu_t \\ \varepsilon_t \end{pmatrix} \tag{1.3}$$

Write in matrix form:

$$\phi_0 Z_t = A_1 Z_{t-1} + u_t \tag{1.4}$$

The (reduced-form) VAR representation:

$$
\begin{aligned}
\begin{pmatrix} \pi_t \\ y_t \\ R_t \end{pmatrix} &= \underbrace{\begin{pmatrix} 1 & -\lambda & 0 \\ 0 & 1 & -b \\ -\kappa & -\omega & 1 \end{pmatrix}^{-1}}_{\phi_0^{-1}} \begin{pmatrix} \beta & 0 & 0 \\ 0 & \theta & 0 \\ 0 & 0 & \rho \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ y_{t-1} \\ R_{t-1} \end{pmatrix} \\
&+ \underbrace{\begin{pmatrix} 1 & -\lambda & 0 \\ 0 & 1 & -b \\ -\kappa & -\omega & 1 \end{pmatrix}^{-1}}_{\phi_0^{-1}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_t \\ \nu_t \\ \varepsilon_t \end{pmatrix}
\end{aligned} \tag{1.5}
$$

Write in matrix form:

$$
Z_t = \phi_0^{-1} A_1 Z_{t-1} + \phi_0^{-1} u_t \tag{1.6}
$$

Structural Vector Autoregressions

- We can try and approximate this model by a VAR:

$$\begin{aligned} \pi_t &= a_0 + a_1\pi_{t-1} + a_2 y_{t-1} + a_3 R_{t-1} + e_{\pi,t} \\ y_t &= b_0 + b_1\pi_{t-1} + b_2 y_{t-1} + b_3 R_{t-1} + e_{y,t} \\ R_t &= c_0 + c_1\pi_{t-1} + c_2 y_{t-1} + c_3 R_{t-1} + e_{R,t} \end{aligned} \tag{1.7}$$

- But the errors are contemporaneously correlated $\longrightarrow$ shocks are not identified:

$$\Sigma = E\left(e_{1,t}, e_{2,t}, e_{3,t}\right) \tag{1.8}$$

- Cannot interpret IRFs and Decompositions

Identification

- The reduced-form VAR as:

$$Z_t = BZ_{t-1} + e_t \tag{1.9}$$

where $Z_t = \{\pi_t, y_t, R_t\}$ and $e_t = \{e_{\pi,t}, e_{y,t}, e_{R,t}\}$

- The structural VAR is written as:

$$\phi_0 Z_t = A_1 Z_{t-1} + u_t \tag{1.10}$$

- Define $A_0 = \phi_0^{-1}$. The link between the structural and reduced-form models is:

$$Z_t = A_0 A_1 Z_{t-1} + A_0 u_t \tag{1.11}$$

- Identification means that we find a matrix A_0 such that $A_0 u_t = e_t$, and $VAR(u_t)$ is diagonal

Identification

- We need to find A_0 such that:
  - $VAR\left(A_0^{-1}e_t\right)$ is diagonal (this is always possible)
  - The new shocks $u_t = A_0^{-1}e_t$ should have an economic interpretation
- How do we find A_0?
  - through the square root of Σ
  - that is $\Sigma = E\left(ee'\right) = A_0'A_0$
  - There are infinite ways of finding this square root $\longrightarrow$ chose method that gives the transformed shocks economic interpretation

Examples of Decomposition

- Recursive identification:

$$\underbrace{\begin{pmatrix} \tilde{A}_{11} & 0 & 0 \\ \tilde{A}_{12} & \tilde{A}_{22} & 0 \\ \tilde{A}_{13} & \tilde{A}_{23} & \tilde{A}_{33} \end{pmatrix}}_{A_0^{-1}} \underbrace{\begin{pmatrix} e_{\pi,t} \\ e_{y,t} \\ e_{R,t} \end{pmatrix}}_{e_t} = \underbrace{\begin{pmatrix} \tilde{A}_{11} e_{\pi,t} \\ \tilde{A}_{12} e_{\pi,t} + \tilde{A}_{22} e_{y,t} \\ \tilde{A}_{13} e_{\pi,t} + \tilde{A}_{23} e_{y,t} + \tilde{A}_{33} e_{R,t} \end{pmatrix}}_{u_t} \tag{1.12}$$

- Which is written more compactly:

$$A_0^{-1} e_t = u_t \tag{1.13}$$

- u_t are uncorrelated and have economic interpretation:
  - Inflation and output do not respond to R_t in the current period
  - Cholesky decomposition with ordering π, y, R identifies a monetary policy shock

Cholesky Decomposition

- Consider the system for impulse response:

$$\begin{pmatrix} \pi_t \\ y_t \\ R_t \end{pmatrix} = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ y_{t-1} \\ R_{t-1} \end{pmatrix} + \underbrace{\begin{pmatrix} A_{11} & 0 & 0 \\ A_{12} & A_{22} & 0 \\ A_{13} & A_{23} & A_{33} \end{pmatrix}}_{A_0} \underbrace{\begin{pmatrix} u_1 \\ u_2 \\ u_{mp} \end{pmatrix}}_{u_t} \tag{1.14}$$

- So the impulse response function in period 1 is:

$$\begin{pmatrix} u_1 \\ u_2 \\ u_{mp} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \longrightarrow \tag{1.15}$$

$$\begin{pmatrix} \pi_1 \\ y_1 \\ R_1 \end{pmatrix} = \begin{pmatrix} A_{11} & 0 & 0 \\ A_{12} & A_{22} & 0 \\ A_{13} & A_{23} & A_{33} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ A_{33} \end{pmatrix} \tag{1.16}$$

- Inflation and output do not respond contemporaneously to the shock

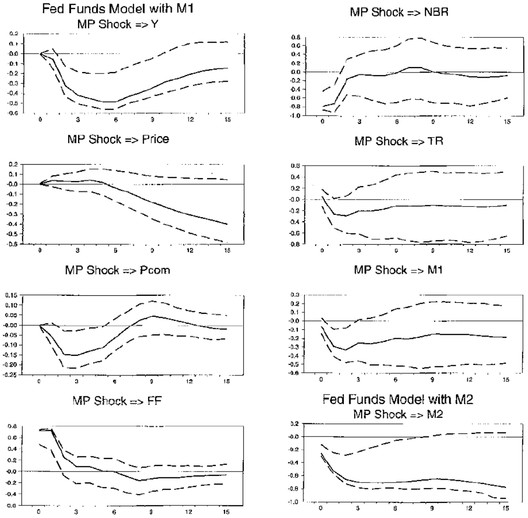
Cholesky Decomposition

- Variable ordered last does not contemporaneously affect the variables ordered before it
- Variable ordered first affects the others contemporaneously

$$\begin{pmatrix} \pi_1 \\ y_1 \\ R_1 \\ H_1 \end{pmatrix} = \begin{pmatrix} A_{11} & 0 & 0 & 0 \\ A_{12} & A_{22} & 0 & 0 \\ A_{13} & A_{23} & A_{33} & 0 \\ A_{14} & A_{24} & A_{34} & A_{44} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ A_{33} \\ A_{34} \end{pmatrix} \tag{1.17}$$

Cholesky Decomposition

The impact of Monetary Policy Shocks in Christiano, Eichenbaum and Evans (1999, Handbook of Monetary Policy, pp. 86)



Examples of Decomposition

- Suppose we have VAR with $\{y, \pi, m, R\}$
- Non-recursive identification

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ \tilde{A}_{12} & 1 & 0 & 0 \\ \tilde{A}_{13} & \tilde{A}_{23} & 1 & \tilde{A}_{43} \\ 0 & 0 & \tilde{A}_{34} & 1 \end{pmatrix}}_{A_0^{-1}} \underbrace{\begin{pmatrix} e_{\pi,t} \\ e_{y,t} \\ e_{M,t} \\ e_{R,t} \end{pmatrix}}_{e_t} = \underbrace{\begin{pmatrix} b_1 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 \\ 0 & 0 & b_3 & 0 \\ 0 & 0 & 0 & b_4 \end{pmatrix}}_{B_0} \underbrace{\begin{pmatrix} u_1 \\ u_2 \\ u_{MD} \\ u_{MS} \end{pmatrix}}_{u_t} \tag{1.18}$$

- Money demand shock:

$$\tilde{A}_{13} e_{\pi,t} + \tilde{A}_{23} e_{y,t} + e_{M_t} + \tilde{A}_{43} e_{R,t} = b_3 u_{MD} \tag{1.19}$$

- Money supply shock:

$$\tilde{A}_{34} e_{M_t} + e_{R,t} = b_4 u_{MS} \tag{1.20}$$

Maximum Likelihood Estimation

- The likelihood function of the VAR is given by:

$$\ln L = \frac{-TN}{2} \ln 2\pi - \frac{T}{2} \ln |A_0 B_0 A_0'| - \frac{1}{2} \sum_{t=1}^{T} (Y_t - X_t B)' (A_0 B_0 A_0')^{-1} (Y_t - X_t B) \tag{1.21}$$

- Maximise this with respect to A_0 and B_0:

Sign Restrictions

- Important new development for structural VARs
- Previous schemes use zero restrictions either in the A_0 or in the long-run
- But DSGE type models imply inequality restrictions
- For example, a monetary policy shock in such a model causes interest rates to rise, and inflation and output to fall
- We can impose these restrictions on the VAR impulse responses for arbitrary number of periods

Sign Restrictions

- Consider imposing restrictions on the contemporaneous period:

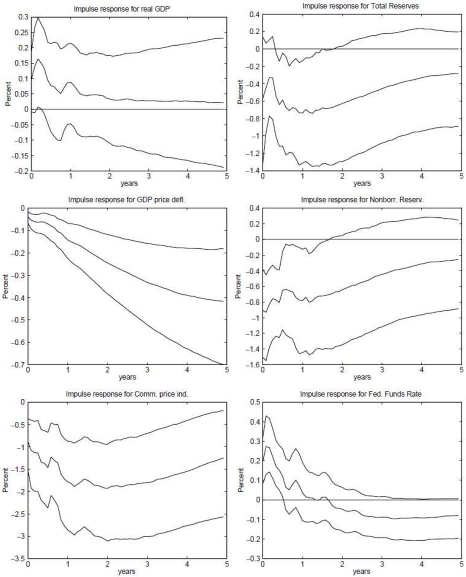
$$\begin{pmatrix} \pi_t \\ y_t \\ R_t \end{pmatrix} = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ y_{t-1} \\ R_{t-1} \end{pmatrix} + \underbrace{\begin{pmatrix} A_{11} & A_{21} & -A_{31} \\ A_{12} & A_{22} & -A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}}_{A_0} \underbrace{\begin{pmatrix} u_1 \\ u_2 \\ u_{mp} \end{pmatrix}}_{u_t} \tag{1.22}$$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_{mp} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \longrightarrow \tag{1.23}$$

$$\begin{pmatrix} \pi_1 \\ y_1 \\ R_1 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{21} & -A_{31} \\ A_{12} & A_{22} & -A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -A_{31} \\ -A_{32} \\ A_{33} \end{pmatrix} \tag{1.24}$$

Sign Restrictions

The impact of Monetary Policy Shocks in Uhlig (2005, JME, pp. 397)



Sign Restrictions

Algorithm – Rubio-Ramirez, Waggoner and Zha (2011, ReStud)

- Step 1: Estimate Σ via OLS
- Step 2: Take the Cholesky decomposition
- Step 3: Find a matrix Q such that $Q'Q = I \longrightarrow$ this is found via a QR decomposition of an $n \times n$ matrix from the normal distribution
- Step 4: Compute the new decomposition as:

$$\bar{A}_0 = A_0 Q \tag{1.25}$$

Note that $\bar{A}_0' \bar{A}_0 = \Sigma$ because $Q'Q = I \longrightarrow$ this is still a square root but with different elements

- Step 5: Check if this $\bar{A}_0$ satisfies restrictions $\longrightarrow$ if it does, we use it to compute IRF etc.

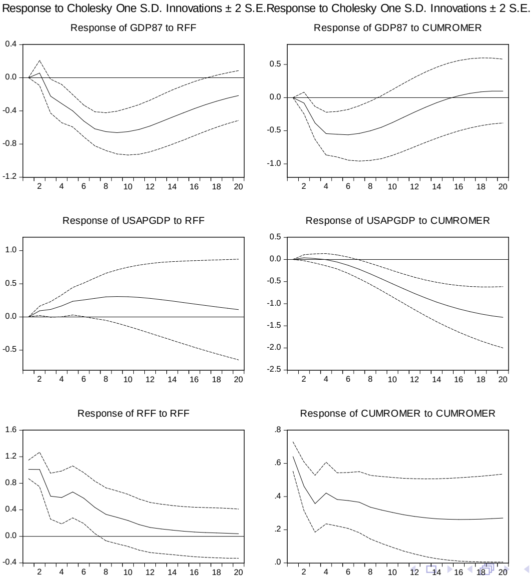
Romer-Romer (2004)

A new measure of monetary policy shocks

- conventional monetary policy measures may be problematic:
  - endogeneity problems: Fed Fund rate tends to rise endogenously with economic activity $\longrightarrow$ standard SVARs might underestimate the negative impact of surprise increases in interest rates on real economic variables
  - anticipatory problems: the Fed typically cuts rates when it sees signs of a future recession is likely. $\longrightarrow$ standard (backward-looking) regressions may fail to account for this
- Romer-Romer aims at extracting the exogenous components of the Fed Funds rate as follows:
  - they regress the changes in the intended funds rate changes around Federal Open Market Committee (FOMC) meetings on "Greenbook" forecasts of inflation and economic activity
  - the residuals from this regression show changes in the intended funds rate not taken in response to information about future economic developments

Romer-Romer (2004)

3-variable VAR(4) for 1969Q2-1995Q2: Romer measure versus Fed Funds Rate



Further topics

Time-varying parameters and stochastic volatility

- The reduced-form VAR as:

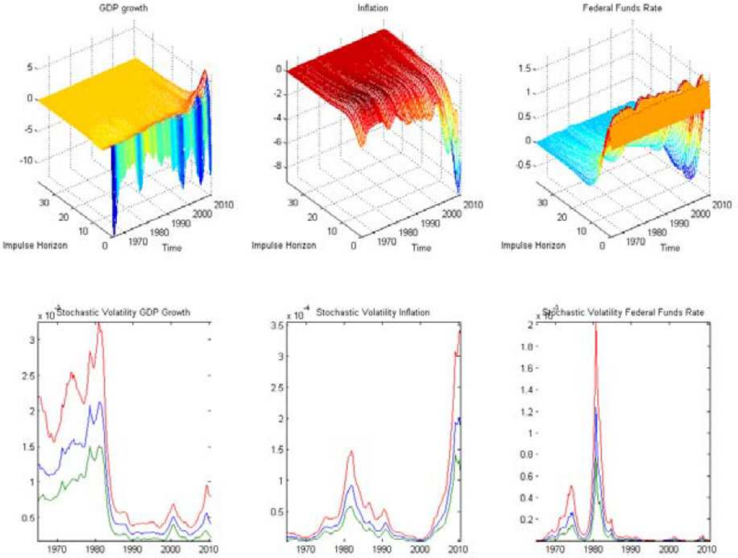
$$Z_t = BZ_{t-1} + e_t \tag{1.26}$$

- So far we assumed constant parameters B and constant variance $\Sigma \longrightarrow$ Is it realistic?
- Change in monetary policy regimes $\longrightarrow$ parameters might have changed over time, B_t
- 'Great Moderation' $\longrightarrow$ volatility of aggregate shocks might have been lower in the 1980s than in the 1970s, Σ_t
- Hence a more general model is:

$$Z_t = B_t Z_{t-1} + e_t, \quad \Sigma_t = E\left(ee'\right) \tag{1.27}$$

Time-varying parameters and stochastic volatility

3-variable monetary VAR in Blake and Mumtaz (2012)



Summary

- VARs can be used for computing:
  - impulse response functions
  - forecast error variance decompositions
  - historical decompositions
- However, we need to identify shocks:
  - Cholesky decompositions
  - Sign restrictions
  - Narrative measures
- More general models should allow time variation in the VAR parameters

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