BANK OF ENGLAND

CENTRE FOR CENTRAL BANKING STUDIES

ECONOMIC MODELLING AND FORECASTING

Recent developments in structural VAR modelling

by

Haroon Mumtaz and Ole Rummel
Centre for Central Banking Studies
Bank of England

10 February 2015

Contents

1 Introduction ... **3**

2 Preparation ... **3**

3 Closed-economy estimation of monetary policy shocks ... **4**

4 Setting up a monetary policy VAR model ... **5**

5 Estimating a monetary policy VAR model ... **5**

6 VAR identification ... **13**

6.1 Imposing short-run restrictions ... 14
6.2 Generating impulse response functions and forecast error variance decompositions ... 19
6.3 Non-recursive identification schemes ... 24
6.4 Imposing long-run restrictions ... 29
6.5 Imposing a historical decomposition ... 34
6.6 Imposing sign restrictions ... 37

References and further reading ... **42**

1 Introduction

The aim of this exercise is to estimate small- and medium-sized vector autoregressions (VAR) for the US; identify structural shocks (such as monetary policy shocks) by imposing appropriate short-run, long-run and sign restrictions using EViews; and assess the results using impulse response functions (IRFs), forecast error variance decompositions (FEVDs) and historical decompositions.

Our data are contained in the EViews workfile **svar_us.wf1**, which includes the following four data series:

- the unemployment rate (`unrate`);
- the nominal M2 money supply measure (`m2`);
- the consumer price index (`cpi`); and
- the federal funds target rate (`ffr`)

Based on this data set, we constructed two other series: the (annual) inflation rate (`inf`, calculated as `(log(cpi) – log(cpi(-12)))*100)`) and the year-on-year growth rate of the nominal M2 money supply measure (`dm2`, calculated as `(log(m2) – log(m2(-12))*100)`).

The data are monthly and span the period from January 1947 (`1947M01` in EViews notation) to December 2008 (`2008M12`), although not all four series are available for such a long period. Unless we are confident in assuming that the underlying monetary policy shocks are robust to different monetary policy regimes (money-supply targeting, exchange-rate targeting, inflation targeting, etc.) and changes in regimes over time, it is important to estimate parameters in structural vector autoregressions (SVARs) on a **single** policy regime. Any regime shift therefore requires a different parameterisation of the SVAR model. This important *caveat* may explain some of the counterintuitive results we will encounter in the following exercises, in which VARs are estimated and SVARs identified over long time periods.[1]

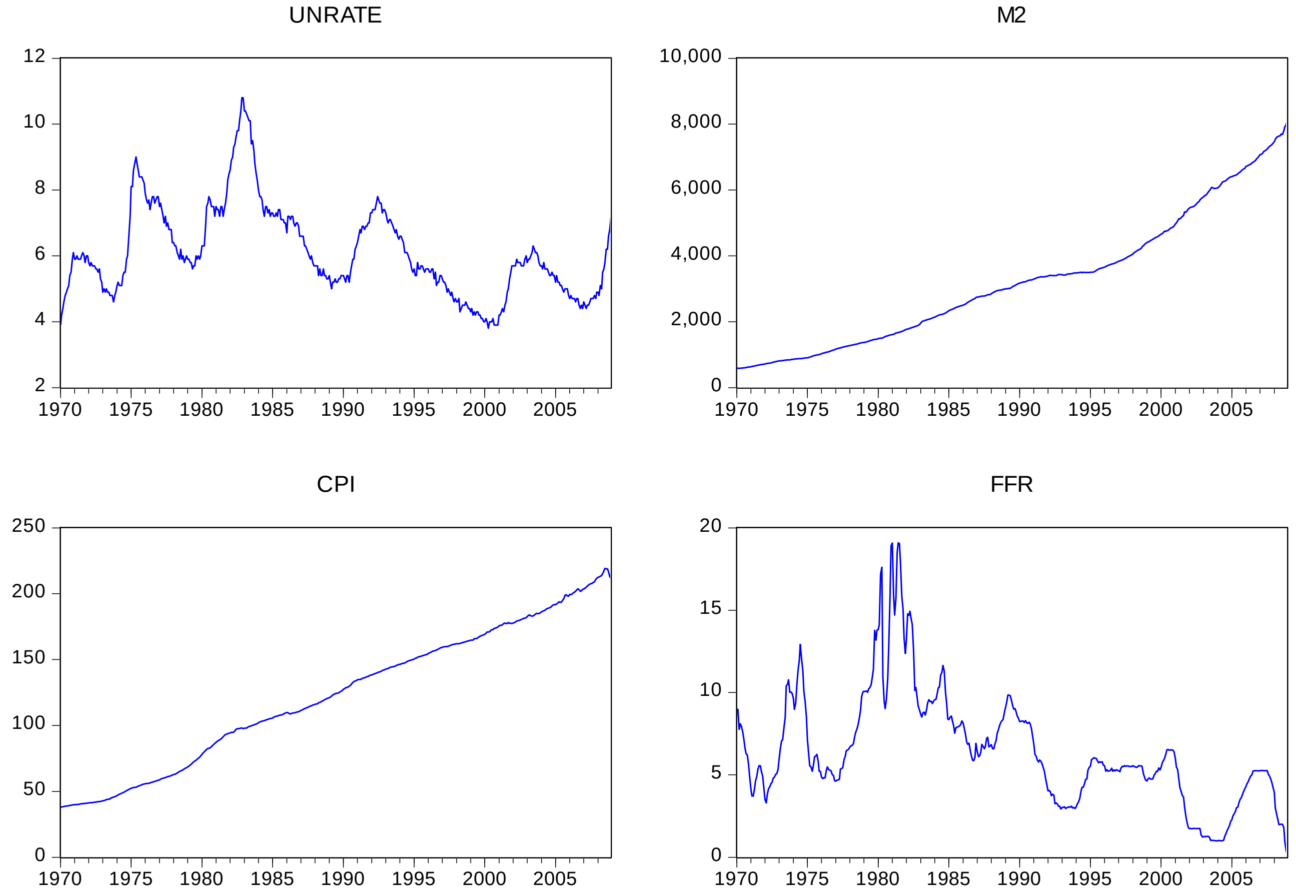
2 Preparations

To open the EViews workfile from within EViews, choose **File**, **Open, EViews Workfile…**, select **svar_us.wf1** and click on **Open**. Alternatively, you can double-click on the workfile icon outside of EViews, which will open EViews automatically.

Whenever we begin working with a new data set, it is always a good idea to take some time to simply examine the data, so the first thing we will do is to plot the data to make sure that it looks fine. This will help ensure that there were no mistakes in the data itself or in the process of reading in the data. It also provides us with a chance to observe the general (time-series) behaviour of the series we will be working with. A plot of our data is shown in Figure 1.

[1] For example, many structural VAR studies of US monetary policy leave out the disinflationary period from 1979 to 1984, which constitutes a different monetary policy regime.

Figure 1: US unemployment rate (`unrate`), nominal M2 money supply (`m2`), consumer price index (`cpi`) and federal funds target rate (`ff`), January 1947 – December 2008



3 Closed-economy estimation of monetary policy shocks

Monetary economics focuses on the behaviour of prices, monetary aggregates, nominal and real interest rates and output. VARs have served as a primary tool in much of the empirical analysis of the interrelationship between those variables and for uncovering the impact of monetary phenomena on the real economy. A summary of the empirical findings in this literature can be found in, *inter alia*, Leeper *et al.* (1996) and Christiano *et al.* (1999).

On the basis of extensive empirical VAR analysis, Christiano *et al.* (1999) derived stylised facts about the effects of a contractionary monetary policy shocks. They concluded that plausible models of the monetary transmission mechanism should be consistent with at least the following evidence on prices, output and interest rates. Following a **contractionary** monetary shock (meaning what?):

(i) the aggregate price level initially responds very little;
(ii) interest rates initially rise; and
(iii) aggregate output initially falls, with a *J*-shaped response, and a zero long-run effect of the monetary policy shock (long-run monetary policy neutrality).

In empirical work, monetary policy shocks are defined as deviations from the monetary policy rule that are obtained by considering an exogenous shock which does not alter the response of the monetary policy-maker to macroeconomic conditions. In other words, the standard VAR approach addresses only the effects of **unanticipated** changes in monetary policy, not the arguably more important effects of the systematic portion of monetary policy or the choice of monetary policy rule.

VAR models of the effects of monetary policy shocks have therefore exclusively concentrated on simulating shocks, leaving the systematic component of monetary policy – as formalised in the monetary policy reaction function or feedback rule – unaltered. In other words, we are in essence carrying out a thought experiment involving an unanticipated monetary policy shock (defined as the residual of the monetary policy reaction function) within an existing monetary policy rule. This exercise is distinct from that of changing the monetary policy rule itself.

4 Setting up a monetary policy VAR model

A key consideration before any estimation can be attempted is the form the variables must have when they enter the VAR: should they enter in levels, gaps or first differences? The answer is simply 'it depends'. It depends on what the VAR is going to be used for. For forecasting purposes, we must avoid potential spurious regressions that may result in spurious forecasts; for the purpose of identifying shocks (such as monetary policy shocks) we have to be careful about the stability of the VAR (whether it can be inverted to yield a corresponding vector moving-average representation), the reliability of our impulse response functions and the statistical properties of the residuals.

Q1. Before we estimate our model, what should we do to ensure unbiased estimates?

Answer: We can use OLS to estimate the VAR, so if we are interested in using the estimated model for forecasting, say, we need to ensure that all variables are either stationary or cointegrated to avoid the spurious regression problem associated with unit roots. For structural identification, on the other hand, we are interested in consistent coefficient estimates as well as the interrelationships between the variables, so we follow Canova (2007, p. 125) and 'To minimise pre-testing problems, we recommend starting by assuming covariance stationarity and deviate from it only if the data overwhelmingly suggest the opposite'. As demonstrated by Sims *et al.* (1990), consistent estimates of VAR coefficients are obtained even when unit roots are present.

Moreover, as shown by Toda and Yamamoto (1995) and Dolado and Lütkepohl (1996), if all variables in the VAR are either I(0) or I(1) and if a null hypothesis is considered that does not restrict elements in each of the parameter matrices A_i's ($i = 1, 2, \ldots, p$), the usual tests have their standard asymptotic normal distributions. Moreover, if the VAR order $p \geq 2$, the t-ratios have their usual asymptotic standard normal distributions, which means that they remain suitable statistics for testing the null hypothesis that a (single) coefficient in one of the parameter matrices is zero (while leaving the other parameter matrices unrestricted). This alleviates the spurious regression problem on the use of standard asymptotic normal distributions.

In light of the results in Sims *et al.* (1990), potential non-stationarity in the VAR under investigation should not affect the model selection process. Moreover, maximum likelihood estimation procedures may be applied to a VAR fitted to the levels even if the variables have unit roots; hence, possible cointegration restrictions are ignored. This is frequently done in (S)VAR modelling to avoid imposing too many restrictions, and we follow this approach here.

5 Estimating a monetary policy VAR model

Having thought about setting up a VAR model for the analysis of monetary policy, we can now proceed to the estimation stage.

Q2. Estimate an unrestricted VAR. We start our estimation with `unrate`, `inf`, `dm2` and `ffr`, including also a constant.

Answer: Use **Quick**, **Estimate VAR…**, enter `unrate`, `inf`, `dm2` and `ffr` in that order into the **Endogenous Variables** box and leave EView's default setting of `1 2` for the lag interval. The sample period for estimation should be `1970M01` to `2008M12`. An equivalent way of getting EViews to do the estimation is to type the following command in the command window:

```
var var01.ls 1 2 unrate inf dm2 ffr
```

This command specifies a VAR with the name **var01** with an initially arbitrary lag length of two. Estimating a VAR generates a lot of output, so Table 1 shows the first and the last two entries of the full EViews VAR(2) output only.

Table 1: Abridged estimation results for the VAR(2) model in `unrate`, `inf`, `dm2` and `ffr`, January 1970 – December 2008

Vector Autoregression Estimates
Sample: 1970M01 2008M12
Included observations: 468
Standard errors in ( ) & t-statistics in [ ]

	UNRATE	INF	DM2	FFR
UNRATE(-1)	0.994464	-0.164703	0.000325	-0.873189
	(0.04831)	(0.09806)	(0.10756)	(0.15409)
	[20.5868]	[-1.67965]	[0.00302]	[-5.66659]
	...	...	...	...
FFR(-1)	-0.036419	0.075281	-0.131009	1.335938
	(0.01341)	(0.02721)	(0.02985)	(0.04276)
	[-2.71662]	[2.76632]	[-4.38882]	[31.2392]
FFR(-2)	0.037999	-0.062557	0.124303	-0.372993
	(0.01342)	(0.02725)	(0.02989)	(0.04282)
	[2.83052]	[-2.29554]	[4.15837]	[-8.70982]
C	0.093708	0.195256	0.111920	0.084761
	(0.03678)	(0.07467)	(0.08190)	(0.11734)
	[2.54755]	[2.61498]	[1.36647]	[0.72237]
R-squared	0.985411	0.985040	0.984490	0.976193
Adj. R-squared	0.985157	0.984779	0.984219	0.975778
Sum sq. resids	13.01948	53.64832	64.55037	132.4843
S.E. equation	0.168419	0.341878	0.375010	0.537249
F-statistic	3875.443	3777.851	3641.798	2352.605
Log likelihood	174.1298	-157.2150	-200.5040	-368.7543
Akaike AIC	-0.705683	0.710321	0.895316	1.614335
Schwarz SC	-0.625905	0.790099	0.975094	1.694113
Mean dependent	6.120085	4.537090	6.644862	6.449444
S.D. dependent	1.382385	2.771110	2.985258	3.451988

Determinant resid covariance (dof adj.)	0.000122
Determinant resid covariance	0.000113
Log likelihood	-529.6955
Akaike information criterion	2.417502
Schwarz criterion	2.736615

Q3. We have selected an arbitrary lag length of 2, but is that appropriate?

Answer: Too short a lag length may result in inconsistent estimates and an inability to capture important dynamics in the data, while too many lags can result in imprecise estimates in small and moderate samples. As such, adding more lags improves the fit but reduces the degrees of freedom and increases the danger of over-fitting. An objective way to decide between these competing objectives is to maximise some weighted measure of these two parameters. This is how the Akaike information criterion (**AIC**), the Schwarz or Bayesian criterion (**SC**) and the Hannan-Quinn criterion (**HQ**) work. These three statistics are measures of the trade-off of improved fit against loss of degrees of freedom, so that the best lag length should **minimise** all of these three statistics.[2]

An alternative to the information criterion is to systematically test for the significance of each lag using a likelihood-ratio test (discussed in Lütkepohl (2005), section 4.3). This is the approach favoured by Sims (1980a), who also suggested a modification to the likelihood-ratio test to take into account small-sample bias. We should follow his recommendation in practice – EViews does as well! Since an unrestricted VAR of lag length p nests the same restricted VAR of lag length $(p - 1)$, the log-likelihood difference multiplied by the number of observations less the number of regressors in the VAR should be distributed as a χ^2-distribution with degrees of freedom equal to the number of restrictions **in the system**, s, i.e., $\chi^2(s)$. In other words:

$$\text{LR} = (T - m)\,\{\log|\Omega_{p-1}| - \log|\Omega_p|\} \sim \chi^2(s)$$

where T is the number of observations, m is the number of parameters estimated per equation (under the alternative) and $\log|\Omega_j|$ is the logarithm of the determinant of the variance-covariance matrix Ω of the VAR with j lags, where $j = 0, 1, 2, \dots, p$. The adjusted test has the same asymptotic distribution as the standard likelihood-ratio test that does not include the adjustment for m, but is less likely to reject the null hypothesis in small samples. For each lag length, if there is no improvement in the fit from the inclusion of the last lag then the difference in errors should not be significantly different from white noise. Make sure that you use the same sample period for the restricted and the unrestricted model, i.e., do not use the extra observation that becomes available when you shorten the lag length.[3] For our example, we use a general-to-specific methodology (some authors specify specific-to-general instead, which I would strongly discourage):[4]

(i) we start with a high lag length (say 18 lags for monthly data);[5]
(ii) for each lag, say p, note the determinant of the residual variance-covariance matrix given in the EViews output and then note the determinant of the residual variance-covariance matrix for a VAR of lag $(p - 1)$;
(iii) take the difference of the logs of the determinants; and
(iv) this difference multiplied by $(T - m)$ should be distributed as a χ^2-distribution with s degrees of freedom.[6]

[2] Some programs maximise the negative of these measures.
[3] EViews will do this automatically.
[4] An informative reference in this regard is Lütkepohl (2007).
[5] Obviously, this will depend on the frequency of your data, so that if you have annual data you could start with 2-3 lags, if you have quarterly data you could start with, say, 10-12 lags, and if you have monthly data with 18-24 lags.
[6] The degrees of freedom will depend on the number of variables as well as the number of lags in the VAR. The total number of variables in a VAR is given by $n(1 + np) = n + n^2p$, where n is the number of variables and p is the number of lags. In our case of a three-variable VAR(5), estimating the model with four lags rather than five means that we have three less parameters to estimate per equation. In total, we will have nine less parameters in the system – in general, n^2. The number of restrictions, s, in the system would therefore be 3*3 = 9.

It is important to note that the residuals from the estimated VAR should be well behaved, that is, there should be no problems with autocorrelation and non-normality. Thus, whilst the AIC, SC or HQ may be good starting points for determining the lag-length of the VAR, it is important to check for autocorrelation and non-normality. If we find that there is autocorrelation for the chosen lag-length, one ought to increase the lag-length of the VAR until the problem disappears. Similarly, if there are problems with non-normality, a useful trick is to add exogenous variables to the VAR (they may correct the problem), including the use of dummy variables and time trends.

At the same time, we should note that the specification of the monetary policy VAR and its statistical adequacy is an issue that has not received much explicit attention in the literature. In most of the applied papers, the lag length is either decided uniformly on an *ad hoc* basis, several different lag lengths are estimated for 'robustness' or the lag length is simply set on the basis of information criteria. Virtually none of the academic papers cited in this exercise undertake any rigorous assessment of the statistical adequacy of the estimated models.

Q4. Test for the appropriate number of lags using the AIC, SC, HQ and the LR information criterion.

Answer: In our VAR window, select **View**, **Lag Structure**, **Lag Length Criteria…**, then enter 18 in the maximum lag specification. The readout should be as in Table 2.

Table 2: VAR lag order selection criteria

VAR Lag Order Selection Criteria
Endogenous variables: UNRATE INF DM2 FFR
Exogenous variables: C
Sample: 1970M01 2008M12
Included observations: 468

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-4159.221	NA	626.4272	17.79154	17.82700	17.80549
1	-721.3427	6802.297	0.000279	3.168131	3.345416	3.237892
2	-529.6955	375.9234	0.000132	2.417502	2.736615*	2.543072*
3	-504.7637	48.47847	0.000127	2.379332	2.840273	2.560711
4	-484.6323	38.80024	0.000125	2.361677	2.964446	2.598864
5	-464.6225	38.22387	0.000123	2.344541	3.089138	2.637537
6	-456.2295	15.88936	0.000127	2.377049	3.263474	2.725854
7	-444.4421	22.11395	0.000129	2.395052	3.423305	2.799665
8	-429.7442	27.32299	0.000130	2.400616	3.570697	2.861039
9	-404.5053	46.48703	0.000125	2.361134	3.673043	2.877365
10	-384.7002	36.14015	0.000123	2.344873	3.798609	2.916912
11	-360.7357	43.32041	0.000119	2.310836	3.906401	2.938685
12	-335.0744	45.94920	0.000114	2.269549	4.006941	2.953206
13	-217.9864	207.6561	7.41e-05	1.837549	3.716769	2.577014
14	-190.9372	47.50942	7.08e-05*	1.790330*	3.811379	2.585605
15	-181.5119	16.39357	7.28e-05	1.818427	3.981304	2.669510
16	-162.3164	33.05891*	7.20e-05	1.804771	4.109476	2.711663
17	-149.7976	21.34614	7.31e-05	1.819648	4.266180	2.782349
18	-137.5898	20.60721	7.44e-05	1.835854	4.424214	2.854364

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

The LR, AIC, SC and HQ information criteria suggest 16, 14, 2 and 2 lags respectively − these are given by the asterisks next to value of the selection criteria associated with each lag length. The 'correct' lag length will depend on the criteria or measure we use. This is typical of these tests and researchers often use the criterion most convenient for their needs. Note that the AIC is inconsistent and **over**estimates the true lag order with positive probability; but that both SC and HQ are consistent. The SC criterion is generally more conservative in terms of lag length than the AIC criterion, i.e., it selects a shorter lag than the other criteria. Ivanov and Kilian (2005) show that while the choice of information criterion depends on the frequency of the data and type of model, HQ is typically more appropriate for quarterly and monthly data.

Q5. Does the chosen VAR have appropriate properties? Are the residuals **stationary, normal** and **not autocorrelated**? Is the VAR **stable**?

A useful tip is to start with the VAR with the minimum number of lags according to the information criteria (in this case 2 lags) and check whether there are problems with autocorrelation and normality.

Answer (autocorrelation): As we are already using a VAR with two lags, there is no need for us to re-estimate the model. For autocorrelation, which is by far the most important problem to rectify, click on **View**, choose **Residual Tests** and pick the **Autocorrelation LM test…**. The output for the VAR(2) model with twelve lags is given in Table 3.

Table 3: VAR(2) residual autocorrelation LM tests up to lag order twelve

VAR Residual Serial Correlation LM Tests
Null Hypothesis: no serial correlation at lag order h
Sample: 1970M01 2008M12
Included observations: 468

Lags	LM-Stat	Prob
1	47.73474	0.0001
2	34.31634	0.0049
3	39.77107	0.0008
4	34.30489	0.0049
5	11.02846	0.8077
6	17.43298	0.3581
7	27.91594	0.0324
8	32.56993	0.0084
9	59.30381	0.0000
10	30.16436	0.0172
11	44.46037	0.0002
12	255.8314	0.0000

Probs from chi-square with 16 df.

There appear to be problems with significant autocorrelation at all but two lags (lags 5 and 6) at conventional significance levels.

To address the problem of **autocorrelation**, try adding further lags to the VAR (having 6 lags in total seems to get rid of the problem of residual autocorrelation at lags 1 and 2, which are the most pressing). This is illustrated in Table 4.

Table 4: VAR(6) residual serial correlation LM tests up to lag order twelve

VAR Residual Serial Correlation LM Tests
Null Hypothesis: no serial correlation at lag order h
Sample: 1970M01 2008M12
Included observations: 468

Lags	LM-Stat	Prob
1	24.63590	0.0765
2	19.91428	0.2241
3	36.47056	0.0025
4	26.90496	0.0426
5	17.51422	0.3531
6	15.86302	0.4626
7	46.24567	0.0001
8	25.61163	0.0597
9	39.13442	0.0010
10	19.40426	0.2483
11	38.21129	0.0014
12	245.9074	0.0000

Probs from chi-square with 16 df.

Adding further lags makes very little discernable difference to the above output. All in all, it is extremely difficult to find a VAR length that eliminates serial correlation at all lag lengths. For that reason, I retain the VAR(6) model, as it is the most parsimonious model that shows no serial correlation at short lags.

Answer (normality): Click on **View**, choose **Residual Tests** and pick **Normality Test…**. For the **normality** test we get the results in Table 5.

Table 5: VAR(6) residual normality tests

VAR Residual Normality Tests
Orthogonalization: Cholesky (Lutkepohl)
Null Hypothesis: residuals are multivariate normal
Sample: 1970M01 2008M12
Included observations: 468

Component	Skewness	Chi-sq	df	Prob.
1	0.219321	3.751935	1	0.0527
2	-0.463215	16.73630	1	0.0000
3	0.236970	4.380074	1	0.0364
4	-1.484939	171.9934	1	0.0000
Joint		196.8617	4	0.0000

Component	Kurtosis	Chi-sq	df	Prob.
1	3.800585	12.49827	1	0.0004
2	7.380420	374.1675	1	0.0000
3	5.413864	113.6214	1	0.0000
4	31.03931	15330.96	1	0.0000
Joint		15831.25	4	0.0000

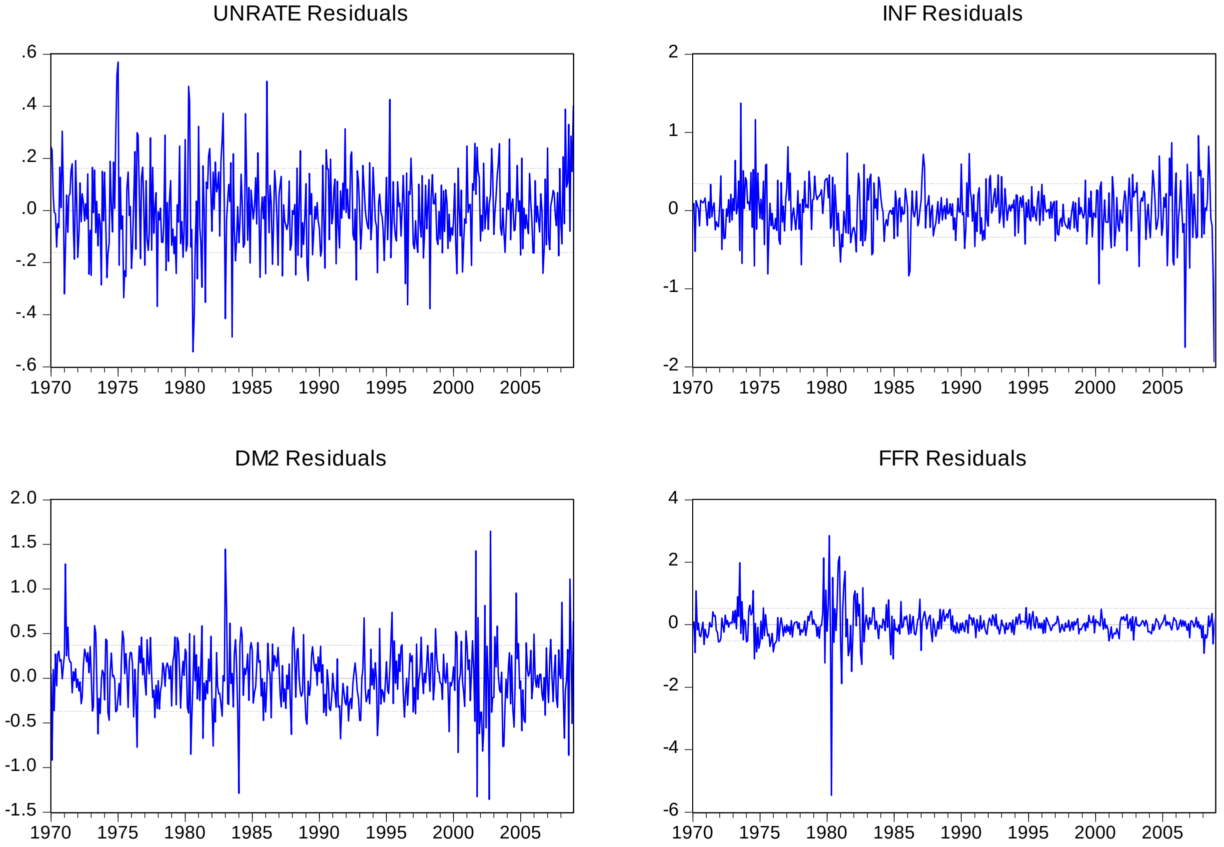
Component	Jarque-Bera	df	Prob.
1	16.25020	2	0.0003

2	390.9038	2	0.0000
3	118.0015	2	0.0000
4	15502.95	2	0.0000
Joint	16028.11	8	0.0000

Thus this VAR does have problems with non-normality (why?). The reason for this is obvious: we note that all but one of the four residual series have problems with skewness, while all four residual series have problems with (excess) kurtosis, leading to the rejection of joint normality for all four residual series.

The non-normality of the residuals results from a number of very large outliers, as can be seen in Figure 2. Very obvious is the disinflationary policy period from 1979 to 1984 in the `ffr` residuals. In addition, inflation (`inf`) shows two very large (negative) residuals towards the end of the sample period. Indeed, looking at the residual diagnostics more closely, we can see the influence of large outliers in the components, as well as the overall Jarque-Bera tests, for all four residual series. This is, unfortunately, a problem. Although normality is not a necessary condition for the validity of many of the statistical procedures related to VAR and SVAR models, deviations from the normality assumption may nevertheless indicate that improvements to the model are possible.[7]

Figure 2: Estimated residual series from the VAR(6) model with `unrate`, `inf`, `dm2` and `ffr`



Rather than EViews' default setting of **Cholesky of covariance (Lütkepohl)** as the **Orthogonalisation method**, some authors prefer to use **Square root of correlation (Doornik-Hendry)**. This is because we must choose a factorisation of the residuals for the multivariate normality test, such that residuals are orthogonal to each other. The approach due to Doornik and Hansen (2008)

[7] One way of dealing with this problem would be to introduce dummies to account for some of the larger outliers.

has two advantages over the one in Lütkepohl (1991, p. 155-158). First, Lütkepohl's test uses the inverse of the lower triangular Cholesky factor of the residual covariance matrix, resulting in a test which is not invariant to a re-ordering of the dependent variables. Second, Doornik and Hansen perform a small-sample correction to the transformed residuals before computing their statistics. We should note that the finding of non-normality is robust to the otrhogonalisation method, though.

Answer (stability): If the VAR is not stable, certain results (such as impulse response standard errors) are not valid. Most importantly, if the VAR is not stable, we will not be able to generate the vector moving-average (VMA) representation from the VAR. In doing this procedure, there will be ($n \times p$) = 24 roots overall, where n is the number of endogenous variables (four) and p is the particular lag length (six). It is easy to check for stability in EViews. Go to **View**, **Lag Structure** and click on **AR Roots Table**. You should get the results in Table 6.

Table 6: Roots of the characteristic polynomial of the VAR(6)

Roots of Characteristic Polynomial
Endogenous variables: UNRATE INF DM2 FFR
Exogenous variables: C
Lag specification: 1 6

Root	Modulus
0.988430 - 0.015979i	0.988559
0.988430 + 0.015979i	0.988559
0.929382	0.929382
0.911611 - 0.093433i	0.916386
0.911611 + 0.093433i	0.916386
0.776831	0.776831
0.551325 + 0.527331i	0.762913
0.551325 - 0.527331i	0.762913
-0.476232 + 0.470089i	0.669164
-0.476232 - 0.470089i	0.669164
-0.659349	0.659349
0.391398 - 0.475514i	0.615878
0.391398 + 0.475514i	0.615878
-0.168878 - 0.578411i	0.602560
-0.168878 + 0.578411i	0.602560
0.130288 + 0.551409i	0.566592
0.130288 - 0.551409i	0.566592
-0.007847 - 0.543865i	0.543922
-0.007847 + 0.543865i	0.543922
-0.425477 - 0.318521i	0.531494
-0.425477 + 0.318521i	0.531494
-0.506107	0.506107
0.327309 + 0.338965i	0.471199
0.327309 - 0.338965i	0.471199

No root lies outside the unit circle.
VAR satisfies the stability condition.

On the plus side, we are finally getting some positive results. The VAR is stable as none of the roots lie outside the unit circle: all the moduli of the roots of the characteristic polynomial are less than one in magnitude. In case one or more roots fall outside the unit circle, adding a time trend (denoted `@trend` in EViews) as an exogenous variable can help the stability of the VAR. Note that adding a time trend will not always correct instability.

We also note that a few of the roots of the characteristic polynomial of the VAR exceed 0.9 in magnitude. We will return to this point later on, but the main argument is that if the characteristic roots are close to one, it will be doubtful if the analytic (asymptotic) confidence intervals that EViews produces will still be accurate. If one or more of the characteristic roots exceed 0.9 in magnitude, we may want to consider bootstrapping the confidence intervals.

Finally, it is worth spending a bit of time investigating the individual coefficient estimates of the VAR(6) we have just estimated. Our small- to medium-sized four-variable VAR with six lags and one deterministic regressor (the constant), which has been estimated on 468 data points, may already be overparameterised. Using the empirical results on the viability of the usual asymptotic distribution of test statistics by Toda and Yamamoto (1995) and Dolado and Lütkepohl (1996), we note that just 18 of the 100 estimated parameters are statistically significant at the 5 per cent level (using the benchmark of the *t*-statistic being larger than 1.96 in absolute magnitude).

6 VAR identification

When Sims (1980a) first advocated the use of a VAR in economics, it was in response to the prevailing orthodoxy at the time that all economic models should be structural models, i.e., that they should include identifying restrictions. Instead, he argued for the use of an unrestricted VAR with no distinction being made in the model between endogenous and exogenous variables. The aim was to free-up econometric modelling from the constraints applied by economic theory and, in effect, to 'let the data speak.'[8]

We are now ready to attempt identification of the structural VAR (SVAR) and, by so doing, attempt to identify monetary policy shocks. In particular, EViews assumes an underlying **structural** equation of the form:

$$Ay_t = C(L)y_t + Bu_t \tag{1}$$

where the structural shocks u_t are normally distributed, i.e., $u_t \sim N(0, \Sigma)$, where Σ is generally assumed to be a diagonal matrix, usually the identity matrix, such that $u_t \sim N(0, I_n)$.[9] Unfortunately, we cannot estimate this equation directly due to identification issues. Instead, we estimate an unrestricted VAR of the form:

$$\begin{aligned} y_t &= A^{-1}C(L)y_t + A^{-1}Bu_t \\ &= H(L)y_t + \varepsilon_t \end{aligned} \tag{2}$$

For reasons outlined in the presentation, the matrices A, B and the C_i's ($i = 1, 2, \dots, p$) are not separately observable from the estimated H_i's and the variance-covariance matrix, $E(\varepsilon_t\varepsilon_t') = \Omega$, of the reduced-form shocks, ε_t. So how can we recover equation (1) from equation (2)? The solution is to impose restrictions on our VAR to identify an underlying structure – but what kind of restrictions are these?

Economic theory can sometimes tell us something about the structure of the system we wish to estimate. As economists, we must convert these structural or theoretical assumptions into feasible restrictions on the VAR. Such restrictions can include, for example:

- a causal (recursive) ordering of shock propagation, e.g., the Cholesky decomposition;
- the fact that nominal variables have no long-run effect on real variables;

[8] But we have already come across at least two necessary restrictions: (i) we need to choose the variables that go into the model and (ii) we have to choose a finite lag length for the VAR.
[9] For reasons having to do with the so-called **AB model** below, I use a different notation from the presentation.

- the long-run behaviour of variables, e.g., the real exchange rate is constant in the long run; and
- the fact that we can derive theoretical restrictions on the signs of the impulse responses resulting from particular structural shocks

There are many types of restrictions that can be used to identify a structural VAR. EViews allows you to impose different types of restrictions. One type imposes restrictions on the short-run behaviour of the system, whereas another type imposes restrictions on the long-run. Using the svarpatterns add-in for **just-identified** SVAR models, EViews allow both types of restrictions to be imposed at the same time, as has been done in Bjørnland and Leitemo (2009), for example. The add-in allows you to impose both short- and long-run restrictions to obtain a non-recursive orthogonalisation of the error terms (as opposed to the recursive Cholesky decomposition) for impulse response analysis that would make more sense from a macroeconomic/structural point of view. In order to use the add-in, you should first estimate a regular VAR model. After that, you can either supply the name of your model or the covariance matrix. The output will be a factor matrix, which can be used further in generating impulse responses (i.e., as a user-specified impulse definition). Starting with EViews 7.2, we can also impose sign restrictions to identify structural VARs, as we will demonstrate further below.

6.1 Imposing short-run restrictions

To impose short run restrictions in EViews, we use equation (2):

$$y_t = A^{-1}C(L)y_t + A^{-1}Bu_t$$

We estimate the random stochastic residual, $A^{-1}Bu_t$, from the residual, ε_t, of the estimated VAR. Comparing the residuals from equations (1) and (2), we find that:

$$\varepsilon_t = A^{-1}Bu_t \tag{3}$$

or, equivalently, that:

$$A\varepsilon_t = Bu_t \tag{3′}$$

In requiring that restriction or identifying schemes must be of the form given by (3′) above, EViews follows what is known as the **AB model**, which is extensively described in Amisano and Giannini (1997). By imposing structure on the matrices A and B, we impose restrictions on the structural VAR in equation (1).

Reformulating equation (3), we have $\varepsilon_t\varepsilon_t' = A^{-1}Bu_tu_t'B'(A^{-1})'$, and, since $\mathrm{E}(u_tu_t') = \mathrm{I}_n$ (the identity matrix) by assumption, we have:

$$\mathrm{E}(\varepsilon_t\varepsilon_t') = \mathrm{E}(A^{-1}Bu_tu_t'B'(A^{-1})') = A^{-1}B\,\mathrm{E}(u_tu_t')\,B'(A^{-1})' = A^{-1}BB'(A^{-1})' = \Omega \tag{4}$$

But can we identify all the elements in A and B from Ω? Equation (4) says that for the n variables in y_t, the symmetry property of the variance-covariance matrix $\mathrm{E}(\varepsilon_t\varepsilon_t') = \Omega$ imposes $n(n + 1)/2$ (identity) restrictions on the $2n^2$ unknown elements in A and B. Thus, an additional $2n^2 – n(n + 1)/2 = (3n^2 – n)/2$ restrictions must be imposed.

In our case, and using the above **AB model**, we have a VAR with four endogenous variables, requiring (3*4*4 – 4)/2 = 22 restrictions. An example of this specification using the Cholesky decomposition identification scheme is:

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 \\ a_{31} & a_{32} & 1 & 0 \\ a_{41} & a_{42} & a_{43} & 1 \end{pmatrix}, B = \begin{pmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{22} & 0 & 0 \\ 0 & 0 & b_{33} & 0 \\ 0 & 0 & 0 & b_{44} \end{pmatrix} \quad (5)$$

where the A matrix by dint of being lower triangular has a recursive structure. Counting restrictions in the A and B matrices above, we have 18 zero restrictions (six in matrix A and twelve in matrix B) as well as another four normalisation restrictions on the diagonal of matrix A, giving us the required total of 22 restrictions. In EViews, these restrictions can be imposed in either matrix form or in text form. Since imposing restrictions in matrix form is relatively straightforward, we will start off by illustrating the text form.[10]

Q6. Impose the Cholesky decomposition, which assumes that shocks or innovations are propagated in the order of `unrate`, `inf`, `dm2` and `ffr`. As we will discuss below, the Cholesky decomposition can be interpreted as a recursive contemporaneous structural model.[11]

Answer: To impose the restriction above in **text format**, we select **Proc** and **Estimate Structural Factorization** from the VAR window menu. In the SVAR options dialog, select **Text** (or **Matrix** as appropriate). Each endogenous variable has an associated variable number, in our example this is:

- `@e1` for the `unrate` residuals;
- `@e2` for the `inf` residuals;
- `@e3` for the `dm2` residuals; and
- `@e4` for the `ffr` residuals

The identifying restrictions are imposed in terms of the ε's, which are the residuals from the reduced-form VAR estimates, and the u's, which are the structural, fundamental or 'primitive' random (stochastic) errors in the structural system. Enter the following in the text box (it is easiest to simply copy the suggested short-run factorisation example from the top of the SVAR options box into the white **Identifying Restrictions** box at the bottom):

```
@e1 = C(1)*@u1
@e2 = C(2)*@e1 + C(3)*@u2
@e3 = C(4)*@e1 + C(5)*@e2 + C(6)*@u3
@e4 = C(7)*@e1 + C(8)*@e2 + C(9)*@e3 + C(10)*@u4
```

The way to interpret these restrictions is that they represent the entries in the $A^{-1}B$ matrix linking ε_t and u_t *via* equation (5), i.e., $\varepsilon_t = A^{-1}Bu_t$.

We make use of the fact that the inverse of a lower (upper) triangular matrix is also a lower (upper) triangular matrix. We can take a closer look at the underlying matrix algebra that results in the set of EViews restrictions. Writing out equation (5) and the identification restrictions (6) in full matrix form, we have:

$$A\varepsilon_t = Bu_t$$

[10] Restrictions using text form are also more flexible, as we can restrict values to be equal.
[11] We should note, however, that most economic theories do not imply recursive contemporaneous systems.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 \\ a_{31} & a_{32} & 1 & 0 \\ a_{41} & a_{42} & a_{43} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \end{bmatrix} = \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{22} & 0 & 0 \\ 0 & 0 & b_{33} & 0 \\ 0 & 0 & 0 & b_{44} \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 \\ a_{31} & a_{32} & 1 & 0 \\ a_{41} & a_{42} & a_{43} & 1 \end{bmatrix}^{-1} \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{22} & 0 & 0 \\ 0 & 0 & b_{33} & 0 \\ 0 & 0 & 0 & b_{44} \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ -a_{21} & 1 & 0 & 0 \\ a_{21}a_{32} - a_{31} & -a_{32} & 1 & 0 \\ a_{21}a_{42} - a_{21}a_{32}a_{43} + a_{31}a_{43} - a_{41} & a_{32}a_{43} - a_{42} & -a_{43} & 1 \end{bmatrix} \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{22} & 0 & 0 \\ 0 & 0 & b_{33} & 0 \\ 0 & 0 & 0 & b_{44} \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \end{bmatrix}$$

$$= \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ -a_{21}b_{11} & b_{22} & 0 & 0 \\ (a_{21}a_{32} - a_{31})b_{11} & -a_{32}b_{22} & b_{33} & 0 \\ (a_{21}a_{42} - a_{21}a_{32}a_{43} + a_{31}a_{43} - a_{41})b_{11} & (a_{32}a_{43} - a_{42})b_{22} & -a_{43}b_{33} & b_{44} \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \end{bmatrix} =$$

$$= \begin{bmatrix} b_{11}u_{1t} \\ -a_{21}b_{11}u_{1t} + b_{22}u_{2t} \\ (a_{21}a_{32} - a_{31})b_{11}u_{1t} - a_{32}b_{22}u_{2t} + b_{33}u_{3t} \\ (a_{21}a_{42} - a_{21}a_{32}a_{43} + a_{31}a_{43} - a_{41})b_{11}u_{1t} + (a_{32}a_{43} - a_{42})b_{22}u_{2t} - a_{34}b_{33}u_{3t} + b_{44}u_{4t} \end{bmatrix}$$

We can see that $\varepsilon_{1t} = b_{11}u_{1t}$, which we can substitute into the remaining three expressions for ε_{2t}, ε_{3t} and ε_{4t}. This results in an expression for ε_{2t} (= $-a_{21}\varepsilon_{1t} + b_{22}u_{2t}$), which we can solve for $b_{22}u_{2t}$ and substitute into the equations for ε_{3t} and ε_{4t}. After a little algebra, we obtain the following four equations in the four unknowns ε_{1t}, ε_{2t}, ε_{3t} and ε_{4t}:

$$\varepsilon_{1t} = b_{11}u_{1t}$$
$$\varepsilon_{2t} = -a_{21}\varepsilon_{1t} + b_{22}u_{2t}$$
$$\varepsilon_{3t} = -a_{31}\varepsilon_{1t} - a_{32}\varepsilon_{2t} + b_{33}u_{3t}$$
$$\varepsilon_{4t} = -a_{41}\varepsilon_{1t} - a_{22}\varepsilon_{2t} - a_{34}\varepsilon_{3t} + b_{44}u_{4t}$$

which you can compare with the four EViews restrictions above. The correspondence between the estimated residuals, ε_t (denoted by `@e` in EViews), and the structural shocks, u_t (denoted by `@u` in EViews) should be obvious, as should the correspondence between `c(1)` (= b_{11}), `c(2)` (= $-a_{21}$), `c(3)` (= b_{22}), `c(4)` (= $-a_{31}$), `c(5)` (= $-a_{32}$), `c(6)` (= b_{33}), `c(7)` (= $-a_{41}$), `c(8)` (= $-a_{22}$), `c(9)` (= $-a_{34}$) and `c(10)` (= b_{44}).

The output after imposing the restrictions on the VAR with six lags and a constant using the **text form** is given in Table 7.

Table 7: Just-identified structural VAR(6) estimates, January 1970 – December 2008 (text form)

Structural VAR Estimates
Sample: 1970M01 2008M12
Included observations: 468
Estimation method: method of scoring (analytic derivatives)
Convergence achieved after 8 iterations
Structural VAR is just-identified

Model: Ae = Bu where E[uu']=I
Restriction Type: short-run text form
@e1 = C(1)*@u1
@e2 = C(2)*@e1 + C(3)*@u2
@e3 = C(4)*@e1 + C(5)*@e2 + C(6)*@u3
@e4 = C(7)*@e1 + C(8)*@e2 + C(9)*@e3 + C(10)*@u4
where
@e1 represents UNRATE residuals
@e2 represents INF residuals
@e3 represents DM2 residuals
@e4 represents FFR residuals

	Coefficient	Std. Error	z-Statistic	Prob.
C(2)	-0.193246	0.097632	-1.979337	0.0478
C(4)	-0.213444	0.105933	-2.014893	0.0439
C(5)	-0.023677	0.049947	-0.474051	0.6355
C(7)	-0.769040	0.144459	-5.323566	0.0000
C(8)	0.081115	0.067834	1.195786	0.2318
C(9)	-0.184540	0.062765	-2.940192	0.0033
C(1)	0.161423	0.005276	30.59412	0.0000
C(3)	0.340942	0.011144	30.59412	0.0000
C(6)	0.368393	0.012041	30.59412	0.0000
C(10)	0.500206	0.016350	30.59412	0.0000
Log likelihood	-507.6145			

Estimated A matrix:

1.000000	0.000000	0.000000	0.000000
0.193246	1.000000	0.000000	0.000000
0.213444	0.023677	1.000000	0.000000
0.769040	-0.081115	0.184540	1.000000

Estimated B matrix:

0.161423	0.000000	0.000000	0.000000
0.000000	0.340942	0.000000	0.000000
0.000000	0.000000	0.368393	0.000000
0.000000	0.000000	0.000000	0.500206

Structural VAR proponents try to avoid overidentifying the VAR structure and propose just enough restrictions to identify the parameters uniquely, which is what we have just done. Note that the EViews output explicitly mentions fact that the structural VAR is just identified. Accordingly, most SVAR models are just identified.

It is always a good idea to consider a recursive solution first, which can serve as a benchmark for later analysis. We should then ask if there is anything unreasonable about the recursive solution, which can be done by looking at the impulse response functions, say. At that stage, we can think about how the system should be modified.

The alternative approach to inputting identifying restrictions would be to use the **matrix form** restrictions. Under this approach, you would create two matrices with the following entries:

$$
\mathrm{A} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ NA & 1 & 0 & 0 \\ NA & NA & 1 & 0 \\ NA & NA & NA & 1 \end{pmatrix}, \mathrm{B} = \begin{pmatrix} NA & 0 & 0 & 0 \\ 0 & NA & 0 & 0 \\ 0 & 0 & NA & 0 \\ 0 & 0 & 0 & NA \end{pmatrix} \quad (6)
$$

Matrices are created by going to **Object**, **New Object…** in the workfile window and selecting **Matrix-Vector-Coef** from the list of possibilities. Make sure to give it an appropriate name – I have called them `matrix_a` and `matrix_b`. Dimension the matrix as required (in our case, we have four columns and four rows). Once the matrix comes up (all entries will be zeros), select **Edit +/-** to access the individual cells in the matrix you have created. We can then enter the individual elements of the first matrix as shown above, consisting of ones, zeros and `NA`'s. Numerical values, such as 0 and 1, set the respective elements of the matrix exactly equal to that value, while `NA`'s tell EViews that these are elemnts of the matrix to be estimated. When you are done, click on **Edit +/-** again, and then close the window. Repeat the exercise for the second matrix.

Once we have created the two matrices they will appear in the list of variables in the workfile. Return to the estimated VAR model, select the **Matrix** specification in the **Structural Factorisation** described above, select **Short-run pattern** and enter the names of the matrices for *A* and *B* as appropriate. Except for the initial change of sign of the estimated coefficients `c(1)` to `c(10)` (which is not carried over to the final representation of the *A* and *B* matrices in the bottom part of Table 8), the resulting output in Table 8 should be identical to the one obtained using the text version in Table 7 above.

Table 8: Just-identified structural VAR(6) estimates, January 1970 – December 2008 (matrix form)

Structural VAR Estimates
Sample: 1970M01 2008M12
Included observations: 468
Estimation method: method of scoring (analytic derivatives)
Convergence achieved after 7 iterations
Structural VAR is just-identified

Model: Ae = Bu where E[uu']=I
Restriction Type: short-run pattern matrix
A =

1	0	0	0
C(1)	1	0	0
C(2)	C(4)	1	0
C(3)	C(5)	C(6)	1

B =

C(7)	0	0	0
0	C(8)	0	0
0	0	C(9)	0
0	0	0	C(10)

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.193246	0.097632	1.979337	0.0478
C(2)	0.213444	0.105933	2.014893	0.0439
C(3)	0.769040	0.144459	5.323566	0.0000
C(4)	0.023677	0.049947	0.474051	0.6355
C(5)	-0.081115	0.067834	-1.195786	0.2318
C(6)	0.184540	0.062765	2.940192	0.0033
C(7)	0.161423	0.005276	30.59412	0.0000
C(8)	0.340942	0.011144	30.59412	0.0000

C(9)	0.368393	0.012041	30.59412	0.0000
C(10)	0.500206	0.016350	30.59412	0.0000
Log likelihood	-507.6145			
Estimated A matrix:				
1.000000	0.000000	0.000000	0.000000	
0.193246	1.000000	0.000000	0.000000	
0.213444	0.023677	1.000000	0.000000	
0.769040	-0.081115	0.184540	1.000000	
Estimated B matrix:				
0.161423	0.000000	0.000000	0.000000	
0.000000	0.340942	0.000000	0.000000	
0.000000	0.000000	0.368393	0.000000	
0.000000	0.000000	0.000000	0.500206	

6.2 Generating impulse response functions and forecast error variance decompositions

Two useful outputs from VARs are the impulse response function (IRF) and the forecast error variance decomposition (FEVD). Impulse responses show how the different variables in the system respond to (identified) shocks, i.e., they show the dynamic interactions between the endogenous variables in the VAR(p) process. Since we have 'identified' the structural VAR, the impulse responses will be depicting the responses to the structural shocks that have an economic interpretation. In other words, once the structural model has been identified and estimated, the effects of the structural shocks, u_t, can be investigated using an impulse response analysis. The latter provides information on the dynamics of the VAR system of equations and how each variable responds and interacts to shocks in the other variables in the system. We do this because the results of the impulse response analysis are often more informative than the parameter estimates of the (S)VAR coefficients themselves.

The same is true for forecast error variance decompositions, which are also popular tools for interpreting VAR models. While impulse response functions trace the effect of a shock to one endogenous variable onto the other variables in the VAR, forecast error variance decompositions (or variance decompositions in short) separate the variation in an endogenous variable into the contributions explained by the component shocks in the VAR. In other words, the variance decomposition tells us the proportion of the movements in a variable due to its 'own' shock versus shocks to the other variables. Thus, the variance decomposition provides information about the relative importance of each (structural) shock in affecting the variables in the VAR. In much empirical work, it is typical for a variable to explain almost all of its own forecast error variance at short horizons and smaller proportions at longer horizons. Such a delayed effect of the other endogenous variables is not unexpected, as the effects from the other variables are propagated through the reduced-form VAR with lags.

Q7. Generate impulse response functions using the Cholesky decomposition following a shock to `ffr`.

Answer: Select **View** and **Impulse Response…**, which opens the **Impulse Responses** menu.[12] On the **Display** tab, select 30 periods, **Multiple Graphs** and **Analytic (asymptotic)** for **Response Standard Errors**. You should enter the variables for which you wish to generate innovations (**Impulses**) and the variables for which you wish to observe the responses (**Responses**). You may either enter the name of the endogenous variables or the numbers corresponding to the ordering of the variables. For example, for the four variables in our VAR (`unrate`, `inf`, `dm2` and `ffr`), you may either type:

[12] Alternatively, click on the **Impulse** button at the top of the VAR box.

`unrate inf dm2 ffr`

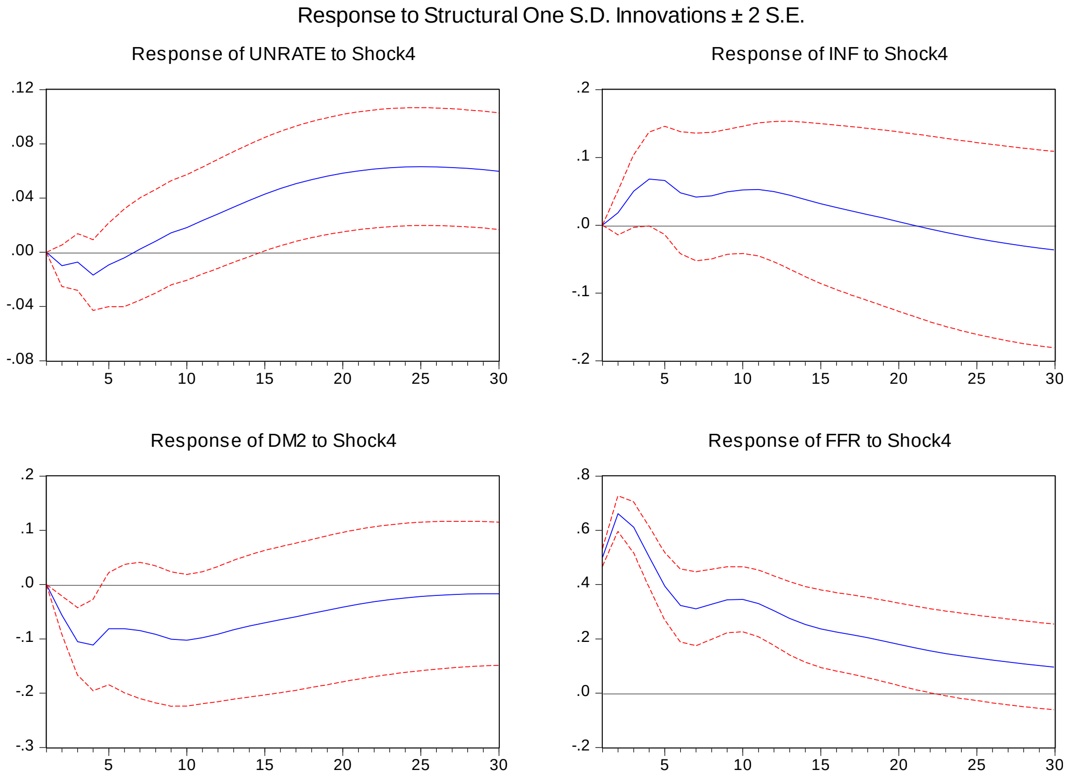
if you wanted to evaluate that particular ordering of the variables or even simply:

1 3 4 2

Note that the four numbers as entered above change the ordering of the variables, as `1 3 4 2` would correspond to `unrate dm2 ffr inf`. But the order in which you enter these variables only affects the display of the results and nothing else.

Type `ffr` (or `4`) in the **Impulses** box and leave the four variables as they are in the **Responses** box (which should correspond to their original order in the estimated VAR). This option will show the impulse response of each variable to a structural shock to `ffr` (or u_{4t}). Remember what this ordering of the four variables implies: the monetary policy shock does not affect the other three variables contemporaneously. The two standard error bands of the impulse response functions are based on **analytical** (or **asymptotic**, i.e., large-sample) results. In small samples, it might be best to bootstrap the standard error bands, which can be easily done in EViews. We will do so in a minute. By clicking on the **Impulse Definition** tab, you will find that the box **Cholesky – dof adjusted** is already chosen for you – this is EViews' default option. Change this option to **Structural Decomposition**. We need to do this as we have just achieved identification by using either the text or the matrix form. Other impulse definitions can be chosen by selecting any of the other options (as we will do later on). Then click on **OK**, which should bring up Figure 3, consisting of the following four charts of impulse response functions.[13]

Figure 3: Impulse response functions of `unrate`, `inf`, `dm2` and `ffr` to a one standard-deviation shock in `ffr` (structural decomposition)



[13] Note that I have adjusted the axes of each chart to make them conform with the later results with bootstrapped standard error bands.

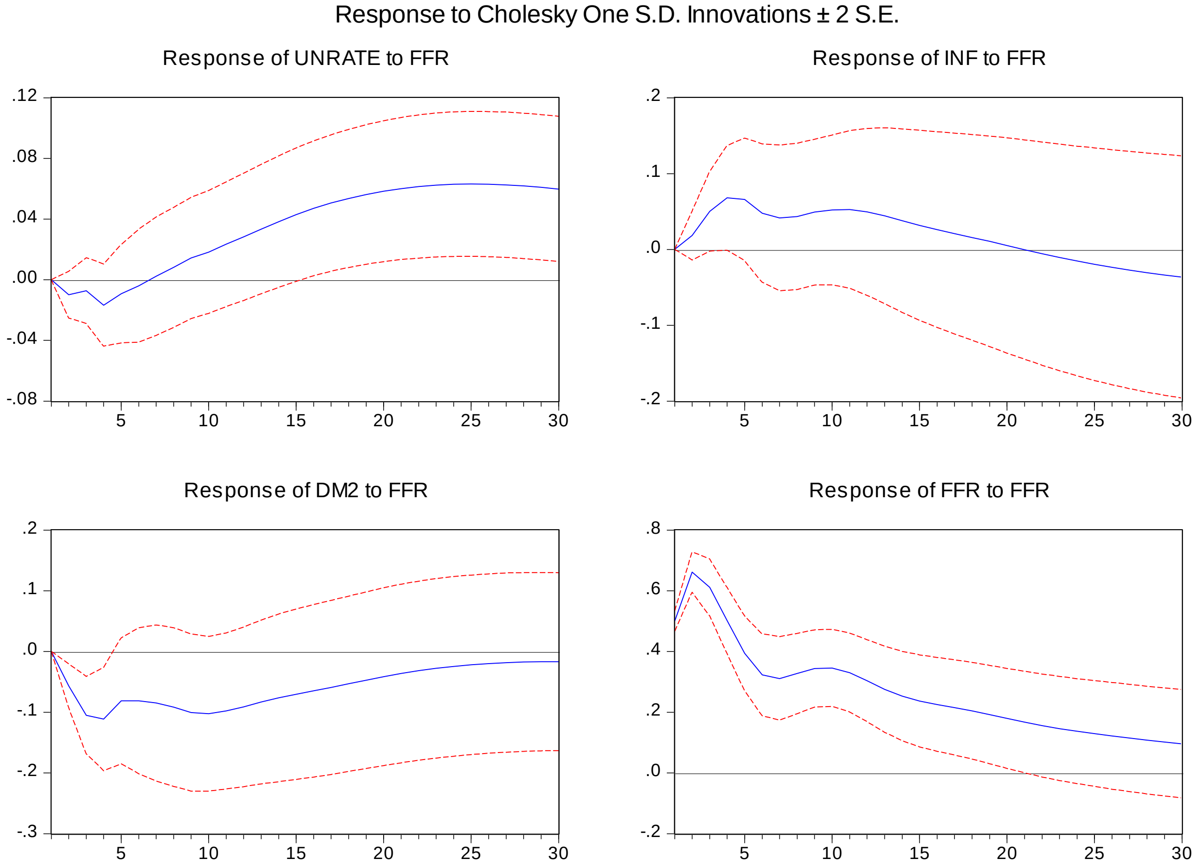
In response to a positive one-standard deviation structural shock to `ffr`, unemployment first falls for four periods before increasing thereafter, inflation increases and is positive for some 20 periods (this is a manifestation of the so-called 'inflation' or 'price' puzzle – we would not expect inflation to **increase** when we **increase** the monetary policy instrument) and the response of the money-supply growth rate is negative over the entire period. Once we add the plus and minus two standard error bands, we can see how significant these effects are. The positive response of the unemployment rate occurs after some 15 months, inflation shows no significant response to `ffr` over all 30 periods, the negative response of `dm2` lasts for some five periods and the shock of `ffr` to itself persists from some 20 periods.[14]

If we limit the analysis to the conventional Cholesky identification scheme, EViews allows you to plot the impulse responses of the Cholesky decomposition without having done that decomposition in the first place. Select **View** and **Impulse Response…**, which opens the **Impulse Responses** menu.[15] On the **Display** tab, select 30 periods and **Multiple Graphs**. In order to illustrate the bootstrapping approach to calculating standard error bands, select **Monte Carlo** with 1,000 repetitions for **Response Standard Errors**. This option shows the impulse response of each variable to shocks to the underlying fundamental shocks (u's). The two standard error bands of the impulse response functions are based on 1,000 Monte Carlo simulations. All the entries for the variables for which you wish to generate innovations (**Impulses**) and the variables for which you wish to observe the responses (**Responses**) should be appropriate, so you do not have to change anything. This option will show the impulse response of each variable to a structural shock to `ffr` (or u_{4t}). By clicking on the **Impulse Definition** tab, you will find that the box **Cholesky – dof adjusted** may already be chosen for you – this is EViews' default option. If not, please select it. Results of this structural factorisation are given in Figure 4 below. As we have carried out the same structural identification scheme in two different ways, the mean impulse responses in Figure 3 should be exactly the same as the mean impulse responses in Figure 3.

[14] Some of the results may be due to the choice as well as number of variables in the VAR. Taking inspiration from Sims (1992), we ought to include a forward-looking variable, such as the exchange rate or a commodity price index, into the VAR. Such a forward-looking asset price can is thought to contain inflationary expectations. This should enlarge the information set of the monetary-policy maker and alleviate the price puzzle. In fact, most industrial-country benchmark VAR models of monetary policy nowadays contain six or seven variables. While some authors have 'improved' their results by adding variables to the VAR, unless those variables are part of the theoretical model the researcher has in mind, it is not clear on what grounds they are selected, other than the fact that they 'work'.

[15] Alternatively, click on the **Impulse** button at the top of the VAR box.

Figure 4: Impulse response functions of `unrate`, `inf`, `dm2` and `ffr` to a one standard-deviation shock in `ffr` (Cholesky decomposition)



In light of Christiano *et al.*'s (1999) three stylised facts about the effects of contractionary monetary policy shocks, let us analyse the impact of a (positive) shock to the short-term interest rate on the variables in the VAR. The short-term interest rate obviously increases as a result of a one-time positive shock to itself (the increase is equal to 0.5010, a value we have come across before as `c(10)`!), but the effect of the monetary shock dies down over time. After some 20 periods, the (one-time) increase in the short-term interest rate is no longer statistically significant. Unemployment shows a *J*-shaped response, and the initial negative effect is not statistically significant. After some 15 periods, unemployment shows a statistically significant positive response, i.e., unemployment rises with a lag in response to a (one-time) increase of the monetary policy rate.[16] The growth rate of M2 is negative for four periods. Whether this corresponds to Christiano *et al.*'s (1999) first stylised fact of the price level initially responding very little is open to debate.

Q8. Generate forecast error variance decompositions for the identification scheme in our example.

Answer: Select **View** and **Variance Decomposition…** as well as the **Table** option, no standard errors and 20 periods. The following table gives the variance decomposition of the three variables in the VAR to the identified structural shocks (at four-quarter intervals to conserve space). Again, EViews allows you to calculate the forecast error variance decomposition using the Cholesky decomposition without having done that decomposition in the first place. Variance decompositions in Table 9 are given

[16] Running the model for longer shows that this positive response is temporary. The unemployment rate becomes insignificantly different from zero after 37 periods.

without standard errors, although you can easily use asymptotic standard errors or bootstrap them using the **Monte Carlo** option.

Table 9: Forecast error variance decompositions

Period	S.E.	UNRATE	INF	DM2	FFR
Variance Decomposition of UNRATE:					
1	0.161423	100.0000	0.000000	0.000000	0.000000
4	0.344465	98.78913	0.103814	0.744764	0.362289
8	0.555928	97.02692	0.996940	1.781263	0.194880
12	0.681005	93.30450	2.848559	3.309395	0.537542
16	0.745788	87.38634	6.074912	4.897978	1.640771
20	0.785678	80.00969	10.50811	6.059658	3.422547
Variance Decomposition of INF:					
Period	S.E.	UNRATE	INF	DM2	FFR
1	0.342366	0.830181	99.16982	0.000000	0.000000
4	0.889197	6.243338	92.06882	0.742325	0.945517
8	1.351391	19.85499	78.34929	0.825352	0.970361
12	1.739667	30.23973	68.27711	0.556082	0.927081
16	2.039289	36.81435	61.82805	0.561084	0.796514
20	2.257000	40.24460	58.13138	0.957761	0.666263
Variance Decomposition of DM2:					
Period	S.E.	UNRATE	INF	DM2	FFR
1	0.370021	0.830285	0.047596	99.12212	0.000000
4	1.140926	1.226387	6.876585	89.85159	2.045438
8	1.763439	6.725110	9.215086	82.27693	1.782871
12	2.159722	11.76674	8.510674	77.70805	2.014531
16	2.380859	14.34219	7.479670	76.13418	2.043960
20	2.495288	14.65542	6.832913	76.48555	2.026118
Variance Decomposition of FFR:					
Period	S.E.	UNRATE	INF	DM2	FFR
1	0.519794	5.369646	0.314396	1.710574	92.60538
4	1.355301	27.54169	0.543350	0.404396	71.51057
8	1.850860	47.34803	0.510600	0.260625	51.88074
12	2.255226	54.61478	1.412893	0.391748	43.58058
16	2.524775	58.50754	2.250183	0.596650	38.64563
20	2.705890	59.99535	3.364363	0.844271	35.79601
Cholesky Ordering: UNRATE INF DM2 FFR					

Of some interest is the effect of nominal on real variables, as a standard result of the SVAR literature is that the monetary policy shock explains a relatively small fraction of the forecast error of real activity measures or inflation. As such, inflation, money-supply growth and the short-term interest rate together predict only a small percentage of the variance of unemployment, equal to some 20 per cent after 20 periods. Note how the variance decomposition of `unrate` due to a shock to itself is still equal to some 80 per cent at the end of the period.

The picture is quite different for inflation. The percentage of variance of inflation explained by `ffr` never goes above 1 per cent. The same is true for `dm2`. On the other hand, the percentage of variance explained by `unrate` increases quite quickly from slightly less than 1 per cent in the first period to some 40 per cent in period 20. Finally, even after 20 periods, inflation still explains the majority (58 per cent) of its own variation.

The other two variance decompositions for `dm2` and `ffr` should be assessed along similar lines.

6.3 Non-recursive identification schemes

Recall that the main requirement of identification is to ensure that we can uniquely recover all the parameters in the A and B matrices from the variance-covariance matrix Ω of the estimated residuals. We do this by imposing the necessary $(3n^2 - n)/2$ additional restrictions. Recall from the presentation that it is only the overall number of identifying restrictions that matters; there is therefore nothing that necessarily requires identification restrictions to follow the Wold causal chain, i.e., a recursive structure. This may a bit too cavalier, though, as the following examples show. We can therefore impose identifying structures different from the recursive one. In the case of the above four-variable VAR (`unrate`, `inf`, `dm2`, `ffr`), we could postulate the following **non-recursive** system for **illustrative** purposes only. In other words, I have spent no time at all trying to come up with an economic rationale for this particular non-recursive identification scheme. The short-run matrix for the structural factorisation is given by `matrix_a_nr1`:

$$\texttt{matrix\_a\_nr1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ NA & 1 & NA & 0 \\ NA & 0 & 1 & 0 \\ NA & NA & NA & 1 \end{pmatrix}, \texttt{matrix\_b2} = \begin{pmatrix} NA & 0 & 0 & 0 \\ 0 & NA & 0 & 0 \\ 0 & 0 & NA & 0 \\ 0 & 0 & 0 & NA \end{pmatrix} \tag{7}$$

Results for this non-recursive identification scheme are given in Table 10 below.

Table 10: Non-recursively identified structural VAR(6) estimates, January 1970 – December 2008 (matrix form)

Structural VAR Estimates
Sample: 1970M01 2008M12
Included observations: 468
Estimation method: method of scoring (analytic derivatives)
Convergence achieved after 7 iterations
Structural VAR is just-identified

Model: Ae = Bu where E[uu']=I
Restriction Type: short-run pattern matrix
A =

1	0	0	0
C(1)	1	C(5)	0
C(2)	0	1	0
C(3)	C(4)	C(6)	1

B =

C(7)	0	0	0
0	C(8)	0	0
0	0	C(9)	0
0	0	0	C(10)

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.197480	0.098016	2.014771	0.0439
C(2)	0.208869	0.105518	1.979461	0.0478
C(3)	0.769040	0.144459	5.323566	0.0000
C(4)	-0.081115	0.067834	-1.195786	0.2318
C(5)	0.020270	0.042760	0.474051	0.6355
C(6)	0.184540	0.062765	2.940192	0.0033
C(7)	0.161423	0.005276	30.59412	0.0000

C(8)	0.340860	0.011141	30.59412	0.0000
C(9)	0.368481	0.012044	30.59412	0.0000
C(10)	0.500206	0.016350	30.59412	0.0000
Log likelihood	-507.6145			
Estimated A matrix:				
1.000000	0.000000	0.000000	0.000000	
0.197480	1.000000	0.020270	0.000000	
0.208869	0.000000	1.000000	0.000000	
0.769040	-0.081115	0.184540	1.000000	
Estimated B matrix:				
0.161423	0.000000	0.000000	0.000000	
0.000000	0.340860	0.000000	0.000000	
0.000000	0.000000	0.368481	0.000000	
0.000000	0.000000	0.000000	0.500206	

While Table 8 shows the results of a structural identification using a recursive order and the Cholesky decomposition, Table 10 shows the results of a structural orthogonalisation using a non-recursive identification scheme. How do we know which one is better? The simple answer is that we cannot choose between them. We note that the maximised value of the log-likelihood for both models is -507.6145, i.e., they are **observationally equivalent**. In other words, we cannot choose between them based on the fit of the data alone, and some other criterion is required. In many cases, this will be the reasonableness of the impulse response functions. (Comparing the impulse response functions between the two identification schemes is left as an optional exercise.)

On the other hand, the non-recursive identification scheme put forward below does have some grounding in economic theory. Assume that data on the price level and the unemployment rate emerge only with a lag and that the monetary policymaker does not respond to changes in inflation and unemployment within the period. In addition, inflation and unemployment do not respond within the period to changes in money supply growth and the monetary policy rate. In the case of the above four-variable VAR (`unrate`, `inf`, `dm2`, `ffr`), we could postulate the following **non-recursive** system:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 \\ a_{31} & a_{32} & 1 & a_{34} \\ 0 & 0 & a_{43} & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \end{pmatrix} = \begin{pmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{22} & 0 & 0 \\ 0 & 0 & b_{33} & 0 \\ 0 & 0 & 0 & b_{44} \end{pmatrix} \begin{pmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \end{pmatrix} \tag{8}$$

The first relationship in matrix (8) shows that the structural shocks to unemployment are equivalent to the estimated reduced-form shocks, or that structural shocks are autonomous of the other variables in the system. If we regard unemployment as a rough monthly proxy of (real) output, the third relationship in system (8) contains contemporaneously all four of the traditional arguments of liquidity preference in an IS-LM model. For that reason, it could be labelled a ‘money demand’ equation. Similarly, the final relationship in matrix (8), which includes the growth rate of the money supply and the short-term interest rate, could be regarded as a ‘money supply’ equation.

Q9. Identify a SVAR with this identification system.

$$A\varepsilon_t = Bu_t$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 \\ a_{31} & a_{32} & 1 & a_{34} \\ 0 & 0 & a_{43} & 1 \end{bmatrix}\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \end{bmatrix} = \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{22} & 0 & 0 \\ 0 & 0 & b_{33} & 0 \\ 0 & 0 & 0 & b_{44} \end{bmatrix}\begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 \\ a_{31} & a_{32} & 1 & a_{34} \\ 0 & 0 & a_{43} & 1 \end{bmatrix}^{-1}\begin{bmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{22} & 0 & 0 \\ 0 & 0 & b_{33} & 0 \\ 0 & 0 & 0 & b_{44} \end{bmatrix}\begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ -a_{21} & 1 & 0 & 0 \\ \dfrac{a_{21}a_{32} - a_{31}}{(1 - a_{34}a_{43})} & -\dfrac{a_{32}}{(1 - a_{34}a_{43})} & \dfrac{1}{(1 - a_{34}a_{43})} & -\dfrac{a_{34}}{(1 - a_{34}a_{43})} \\ \dfrac{a_{31}a_{43} - a_{21}a_{32}a_{43}}{(1 - a_{34}a_{43})} & \dfrac{a_{32}a_{43}}{(1 - a_{34}a_{43})} & -\dfrac{a_{43}}{(1 - a_{34}a_{43})} & 1 \end{bmatrix}\begin{bmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{22} & 0 & 0 \\ 0 & 0 & b_{33} & 0 \\ 0 & 0 & 0 & b_{44} \end{bmatrix}\begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \end{bmatrix}$$

$$= \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ -a_{21}b_{11} & b_{22} & 0 & 0 \\ \dfrac{(a_{21}a_{32} - a_{31})b_{11}}{(1 - a_{34}a_{43})} & -\dfrac{a_{32}b_{22}}{(1 - a_{34}a_{43})} & \dfrac{b_{33}}{(1 - a_{34}a_{43})} & -\dfrac{a_{34}b_{44}}{(1 - a_{34}a_{43})} \\ \dfrac{(a_{31}a_{43} - a_{21}a_{32}a_{43})b_{11}}{(1 - a_{34}a_{43})} & \dfrac{a_{32}a_{43}b_{22}}{(1 - a_{34}a_{43})} & -\dfrac{a_{43}b_{33}}{(1 - a_{34}a_{43})} & b_{44} \end{bmatrix}\begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \end{bmatrix} =$$

$$= \begin{bmatrix} b_{11}u_{1t} \\ -a_{21}b_{11}u_{1t} + b_{22}u_{2t} \\ \dfrac{(a_{21}a_{32} - a_{31})b_{11}u_{1t} - a_{32}b_{22}u_{2t} + b_{33}u_{3t} - a_{34}b_{44}u_{4t}}{(1 - a_{34}a_{43})} \\ \dfrac{(a_{31}a_{43} - a_{21}a_{32}a_{43})b_{11}u_{1t} + a_{32}a_{43}b_{22}u_{2t} - a_{34}b_{33}u_{3t} + b_{44}u_{4t}}{(1 - a_{34}a_{43})} \end{bmatrix}$$

Answer: Algebraically, the matrix contains one more than the required $n(n-1)/2 = (4)(3)/2 = 6$ zero restrictions required for identification. We thus have one over-identifying restriction.

These restrictions are most easily imposed using the matrix form. We therefore need to create the two matrices *A* and *B* by hand as described above.[17] Once you have created the two new matrices, go back to the VAR window, select **Proc**, **Estimate Structural Factorisation...**, select **Matrix**, **Short-run pattern** and enter the two matrices where appropriate. Click on **OK** to estimate the elements of the two matrices.

While only three of the five elements of the *A* matrix are statistically significant at the 5 per cent level of significance, the single over-identifying restriction cannot be rejected, as shown in Table 11.

[17] I have called them `matrix_a_nr2` and `matrix_b2` in the workfile.

Table 11: Non-recursively identified structural VAR(6) estimates, January 1970 – December 2008 (matrix form)

Structural VAR Estimates
Sample: 1970M01 2008M12
Included observations: 468
Estimation method: method of scoring (analytic derivatives)
Convergence achieved after 17 iterations
Structural VAR is over-identified (1 degrees of freedom)

Model: Ae = Bu where E[uu']=I
Restriction Type: short-run pattern matrix

A =

1	0	0	0
C(1)	1	0	0
C(2)	C(3)	1	C(5)
0	0	C(4)	1

B =

C(6)	0	0	0
0	C(7)	0	0
0	0	C(8)	0
0	0	0	C(9)

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.193246	0.097632	1.979337	0.0478
C(2)	1.355913	0.537859	2.520944	0.0117
C(3)	-0.110173	0.118637	-0.928655	0.3531
C(4)	-3.875257	2.286923	-1.694529	0.0902
C(5)	1.565775	0.649131	2.412111	0.0159
C(6)	0.161423	0.005276	30.59412	0.0000
C(7)	0.340942	0.011144	30.59412	0.0000
C(8)	0.825854	0.295173	2.797865	0.0051
C(9)	1.578222	0.801250	1.969700	0.0489

Log likelihood	-507.9676			
LR test for over-identification:				
Chi-square(1)	0.706235		Probability	0.4007

Estimated A matrix:

1.000000	0.000000	0.000000	0.000000
0.193246	1.000000	0.000000	0.000000
1.355913	-0.110173	1.000000	1.565775
0.000000	0.000000	-3.875257	1.000000

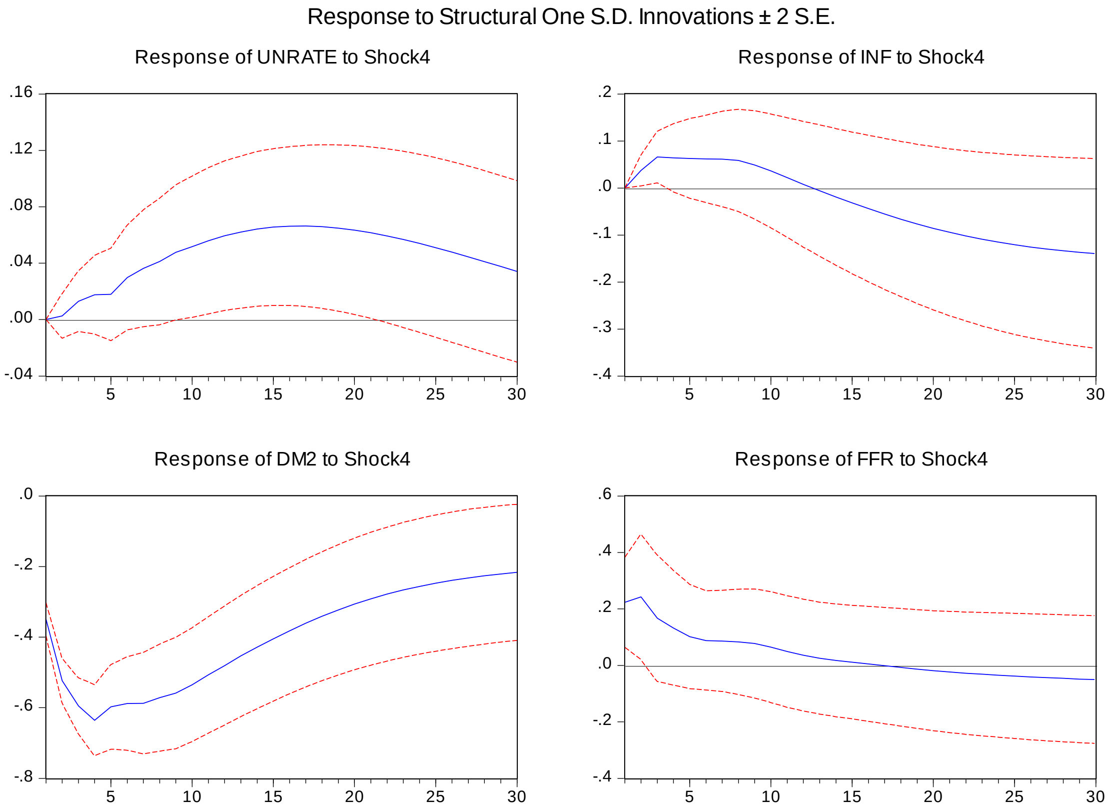
Estimated B matrix:

0.161423	0.000000	0.000000	0.000000
0.000000	0.340942	0.000000	0.000000
0.000000	0.000000	0.825854	0.000000
0.000000	0.000000	0.000000	1.578222

Q10. How do the results for the impulse responses compare to a Cholesky decomposition?

Answer: Click on **Impulse** to bring up the Impulse Responses window. On the **Display** tab, select 30 periods, **Multiple Graphs** and **Analytic (asymptotic)** for **Response Standard Errors**. You should enter the variables for which you wish to generate innovations (**Impulses**) and the variables for which you wish to observe the responses (**Responses**). Type `ffr` (or `4`) in the **Impulses** box and leave the four variables as they are in the **Responses** box. By clicking on the **Impulse Definition** tab, you will find that the box **Cholesky – dof adjusted** is already chosen for you – this is EViews' default option. Change this option to **Structural Decomposition**. We need to do this as we have just achieved identification by using either the text or the matrix form based on a non-recursive, i.e., non-Cholesky decomposition. Other impulse definitions can be chosen by selecting any of the other options (as we will do later on). Then click on **OK**, which should bring up Figure 5, consisting of the following four charts of impulse response functions.

Figure 5: Impulse response functions of `unrate`, `inf`, `dm2` and `ffr` to a one standard-deviation shock in `ffr` (non-recursive decomposition)



Very broadly speaking, the shapes of the new impulse responses are comparable to those in Figures 3 and 4. But some differences emerge. To begin with, `unrate` responds with a lag to the shock in `ffr`, but the response is very short-lived (between periods 10 and 21). Inflation again increases, with a short-lived significant increase up to period 4. The money-supply growth rate shows a much larger (more negative) and significant response over the entire period under observation. Finally, the shock of `ffr` to itself is now much shorter-lived: the lower standard error band crosses the zero line after two periods, meaning the impulse response function is not statistically different from zero after that point.

Let me return to the earlier point that it is only the overall number of identifying restrictions that matters; which might be rather careless or excessively loose. Consider the following non-recursive identification scheme:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & a_{23} & a_{24} \\ a_{31} & a_{32} & 1 & 0 \\ 0 & a_{33} & a_{43} & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \end{pmatrix} = \begin{pmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{22} & 0 & 0 \\ 0 & 0 & b_{33} & 0 \\ 0 & 0 & 0 & b_{44} \end{pmatrix} \begin{pmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \end{pmatrix} \tag{9}$$

Algebraically, system (9) again contains the required number (6) of zero restrictions required for identification. This system implies that for the first two equations:

$$\varepsilon_{1t} = b_{11}u_{1t}$$
$$\varepsilon_{2t} = -a_{23}\varepsilon_{3t} - a_{24}\varepsilon_{4t} + b_{22}u_{2t}$$

whence it follows that the covariance between ε_{1t} and ε_{2t} is equal to zero ($\text{cov}(\varepsilon_{1t}, \varepsilon_{2t}) = 0$). The recursive system discussed above essentially determines the six unknown parameters using the six elements in the variance-covariance matrix of ε_t. But in the alternative A matrix in equation (9), one element of the variance-covariance matrix is equal to zero. i.e., it contains no information about the parameters. This leaves us with just five equations – the non-zero elements in $\text{cov}(\varepsilon_t)$ – to find six parameters. As will be apparent when you ask EViews to carry out the structural factorisation using the short-run matrices `matrix_a_nr2` (which needs to be created from equation (9) above) and `matrix_b`, this cannot be done and EViews will be unable to provide an estimate.

In summary, while there are some advantages to abandoning the recursive assumption, there is also a substantial cost: a broader set of economic relations must be identified. Moreover, even sensible economic models specified by a non-recursive short-run matrix A may not be estimatable. The latter fact is not due to any economic theory reasons, but to the violation of simple computational constraints.

6.4 Imposing long-run restrictions

A second type of restriction that can be imposed in EViews is a long-run restriction. This type of restriction was introduced by Blanchard and Quah (1989). Their restriction scheme is to consider the vector moving-average representation and thus its impulse responses in detail, concentrating on the impact that shocks have on the long-run of variables. Since the long-run is considered, the variables that enter the VAR have to be stationary. Note that if some of the variables are I(1), then it is possible, if other variables are I(0), to decompose the I(1) variable into two components: a permanent and a transitory component. Thus, the Blanchard and Quah decomposition is an alternative form of conducting Beveridge and Nelson (1981) decompositions. Long-run restrictions are not often used to identify monetary policy shocks with the type of variables that we have. Consider our structural VAR from equation (1):

$$y_t = A^{-1}C(L)y_t + A^{-1}Bu_t$$

Rearrangement of this equation yields:

$$y_t = (\mathrm{I} - A^{-1}C(L))^{-1}A^{-1}Bu_t \tag{10}$$

Equation (10) shows how the random (stochastic) shocks affect the long-run levels of the variables. If we define a matrix $M = (\mathrm{I} - A^{-1}C(L))^{-1}A^{-1}B$, the aggregate effect of a shock u is given by matrix M. Hence, if we assume that the (long-run) cumulative effect of a sub-shock u_i on a variable y_j is zero, then the column i and row j element of matrix M should be zero. For example, suppose you have a two-variable VAR where you want to restrict the long-run response of the first endogenous variable, y_{1t}, to the second structural shock, u_{2t}, to be zero, then $m_{12} = 0$:

$$\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} m_{11} & 0 \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} \tag{11}$$

In turn, knowing the values of the matrix *M* tells us something about matrices *A* and *B*. Due to the number of overall restrictions required, EViews also imposes the (necessary) restriction that matrix *A* is the identity matrix.[18] It then uses matrix *M* to estimate matrix *B*.

Long-run restrictions in EViews can be specified either in matrix form (where the matrix *M* is entered) or in text form. We will consider a bivariate VAR using the rate of change of unemployment and the money growth rate. We then impose the long-run restriction that only demand shocks have permanent effects on unemployment and estimate the impulse response function associated with this identification scheme.

Q11. Estimate a reduced-form VAR(6) with `d(unrate)` and `dlog(m2)` over the period from 1990 M1 to 2008 M12, impose the long-run restriction that only demand shocks have permanent effects on unemployment and estimate the impulse response functions of the identification scheme..

Answer: Recall that variables have to be in stationary form in order to impose the long-run identification scheme. Thus we need to transform the two variables to stationary form. The variables we shall use are the first difference of the unemployment rate (`d(unrate)`) and the **monthly** money growth variable (`dlog(m2)`). A VAR with three lags using the first difference of the unemployment rate and monthly money growth as well as a constant appears to get rid of the problem of autocorrelation. The VAR is stable, but the residuals are non-normal, principly on account of the second series (this is left as an optional exercise). The new estimation results are given in Table 12.

Table 12: VAR(3) estimation results for `d(unrate)` and `dlog(m2)`, 1990 M01 – 2008 M12

Vector Autoregression Estimates
Sample: 1990M01 2008M12
Included observations: 228
Standard errors in ( ) & t-statistics in [ ]

	D(UNRATE)	DLOG(M2)
D(UNRATE(-1))	-0.075993	0.003163
	(0.06579)	(0.00145)
	[-1.15505]	[2.18745]
D(UNRATE(-2))	0.217409	0.004986
	(0.06555)	(0.00144)
	[3.31650]	[3.46096]
D(UNRATE(-3))	0.261286	-0.000725
	(0.06804)	(0.00150)
	[3.84041]	[-0.48480]
DLOG(M2(-1))	5.272598	0.262841
	(3.15183)	(0.06926)
	[1.67287]	[3.79478]
DLOG(M2(-2))	-1.187173	0.144012
	(3.24317)	(0.07127)
	[-0.36605]	[2.02063]
DLOG(M2(-3))	2.308013	0.178521
	(3.17300)	(0.06973)
	[0.72739]	[2.56020]

[18] The expression for the long-run response in equation (10) involves the inverse of *A*. Since EViews requires all restrictions to be linear in the elements of *A* and *B*, the *A* matrix must be the identity matrix when you specify a long-run restriction.

C	-0.019956	0.001730
	(0.01825)	(0.00040)
	[-1.09358]	[4.31423]
R-squared	0.143060	0.276817
Adj. R-squared	0.119794	0.257183
Sum sq. resids	4.101135	0.001981
S.E. equation	0.136225	0.002994
F-statistic	6.149050	14.09889
Log likelihood	134.5433	1005.005
Akaike AIC	-1.118801	-8.754432
Schwarz SC	-1.013514	-8.649146
Mean dependent	0.007895	0.004160
S.D. dependent	0.145199	0.003473
Determinant resid covariance (dof adj.)		1.65E-07
Determinant resid covariance		1.55E-07
Log likelihood		1140.205
Akaike information criterion		-9.878991
Schwarz criterion		-9.668417

We now have to create the long-run impact matrix.

$$\begin{pmatrix} d(unrate_t) \\ d\log(m2_t) \end{pmatrix} = \begin{pmatrix} m_{11} & 0 \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} demand\_shock_t \\ supply\_shock_t \end{pmatrix}$$

Once again, imposing this (long-run) restriction can be done either in text or matrix format. To impose this restriction using the text format type:

```
@lr1(@u2)=0
```

To impose the restriction using the matrix format, create the long-run matrix, which I have called `matrix_lr` in the workfile:

$$M = \begin{pmatrix} m_{11} & 0 \\ m_{21} & m_{22} \end{pmatrix} = \begin{pmatrix} NA & 0 \\ NA & NA \end{pmatrix}$$

In other words, the long-run response of the first variable (real output) to the second structural shock (a supply shock) is zero. The output can be found in Table 13.

Table 13: Structural factorisation of SVAR(3) model for `d(unrate)` and `dlog(m2)`, 1990 M01 – 2008 M12 (long-run restrictions)

Structural VAR Estimates
Sample: 1990M01 2008M12
Included observations: 228
Estimation method: method of scoring (analytic derivatives)
Convergence achieved after 8 iterations
Structural VAR is just-identified

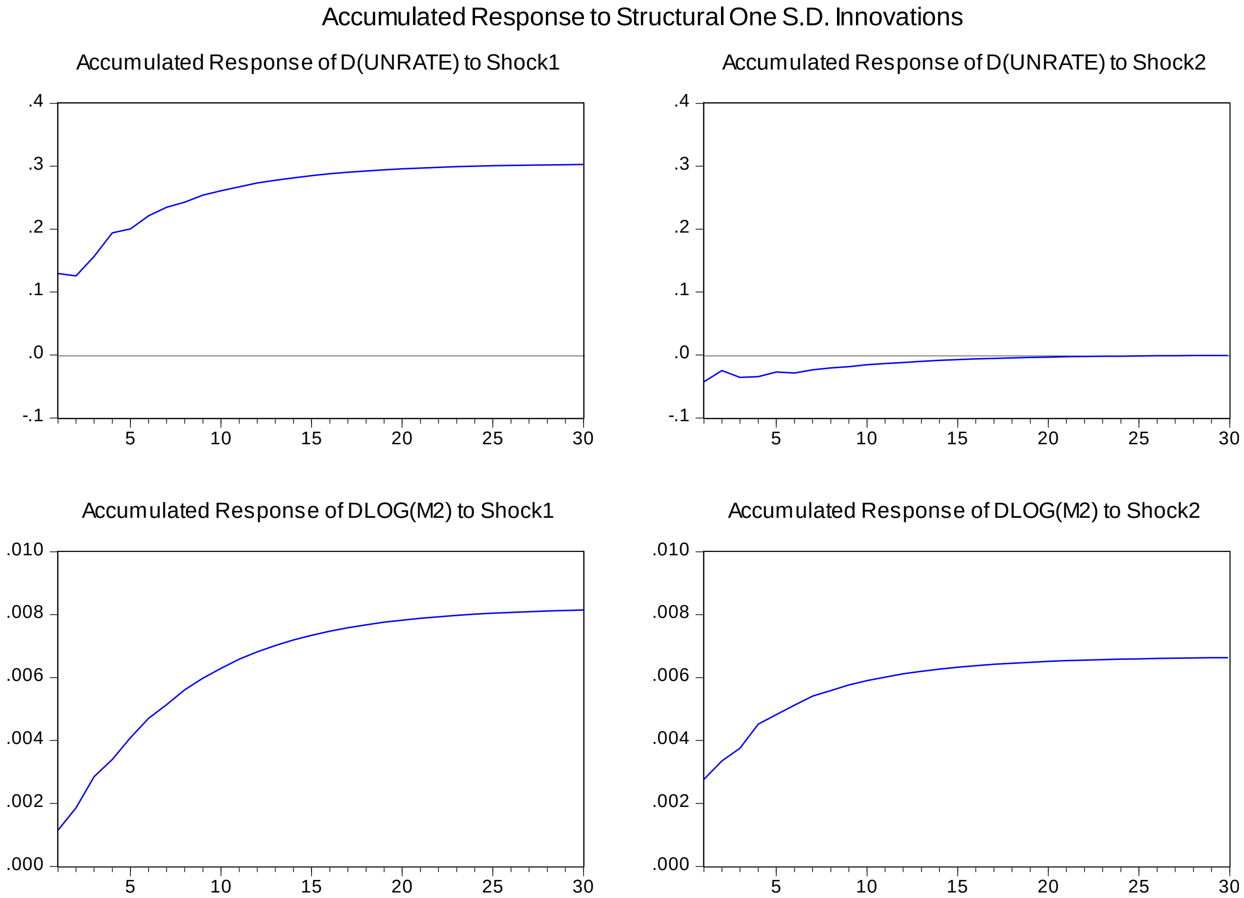
Model: Ae = Bu where E[uu']=I
Restriction Type: long-run pattern matrix
Long-run response pattern:

C(1)	0			
C(2)	C(3)			

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.304681	0.014268	21.35416	0.0000
C(2)	0.008227	0.000586	14.04024	0.0000
C(3)	0.006667	0.000312	21.35416	0.0000
Log likelihood	1133.095			
Estimated A matrix:				
1.000000	0.000000			
0.000000	1.000000			
Estimated B matrix:				
0.129385	-0.042623			
0.001149	0.002764			

In terms of impulse response functions, we are now no longer considering a Cholesky decomposition. In order to tell EViews this, we have to go to **View/Impulse Responses...**, check the box for **Accumulated Responses** and select the **Structural Decomposition** option on the **Impulse Definition** tab. Note that EViews does not give a name to the shocks but labels them sequentially. In our case, the shocks are referred to as `Shock1` and `Shock2`. In addition, it is not possible to get EViews to calculate standard error bands around the impulse response functions automatically. The (accumulated – why?) impulse responses associated with this identification are given in Figure 6.

Figure 6: Accumulated impulse response functions



Shock1 is the demand shock, whereas Shock2 is the supply shock. Thus, a demand shock increases unemployment (as expected?) and monthly money growth. A supply shock leaves output unaffected in the long-run (as it should!) and increases monthly money growth.

From the estimated structural VAR, how can we generate the fundamental shocks, i.e., the u_t's in equation (1), using EViews? To generate the fundamental shocks, we use the equation $A\varepsilon_t = Bu_t$, where ε_t is the error or residual from the VAR regression which has been generated, and matrices A and B come from the estimated structural VAR. The fundamental shocks are then simply: $\hat{u}_t = B^{-1}A\varepsilon_t$.

Q12. Generate the fundamental shocks from the long-run SVAR.

Answer: Select **Proc** and **Make Residuals**. EViews will automatically generate series named resid?? in the same ordering as the VAR estimate. In this case we assume that they are resid0 and resid02. Change the names as appropriate – I have called them lr_shock1 and lr_shock2.

Create two matrices, calling them mat_lr_a and mat_lr_b by typing in the command window:

```
matrix(2,2) mat_lr_a
matrix(2,2) mat_lr_b
```

and enter the estimated coefficient values from the estimated long-run SVAR, i.e.:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

for the entries of mat_lr_a (remember that EViews assumes the A matrix to be the identity matrix, cf. Footnote 18) and:

$$\begin{pmatrix} 0.129385 & -0.042623 \\ 0.01149 & 0.002764 \end{pmatrix}$$

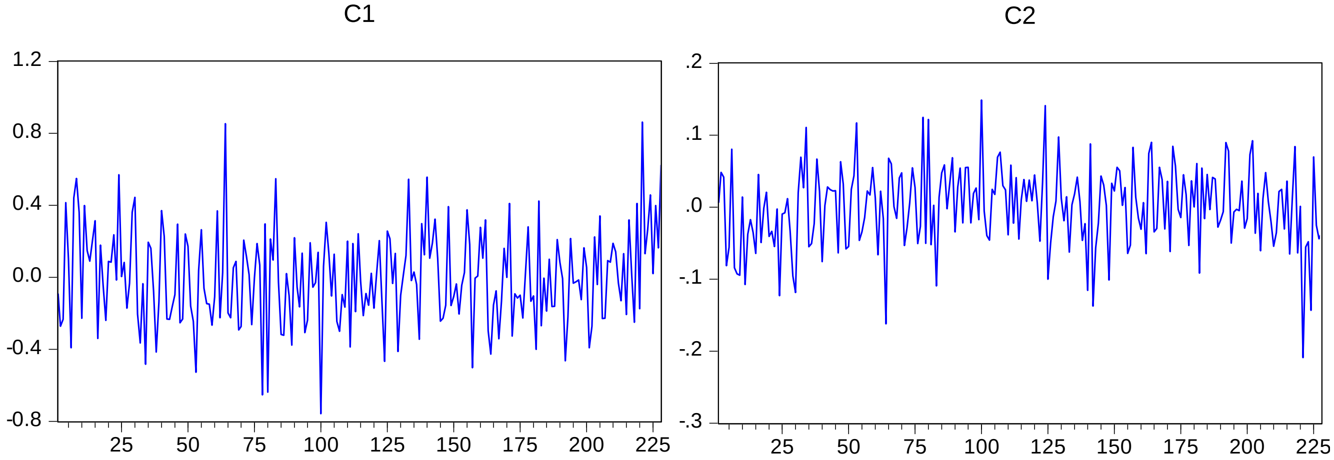
for mat_lr_b. These number come from Table 13 above.

Type in the command window (or create a programme and run it):

```
group resgroup lr_shock1 lr_shock2
matrix resmatrix = @convert(resgroup)
matrix resfund = @transpose(@inverse(mat_lr_b) * mat_lr_a *
@transpose(resmatrix))
show resfund
```

In the resfund matrix object that will open automatically, select **View, Graph, Line** to see a plot of the fundamental structural shocks (Figure 7).

Figure 7: Fundamental shocks



6.5 Imposing a historical decomposition

The forecast error variance decomposition is widely employed as a useful analytical tool for the *ex post* analysis of (S)VARs. As illustrated in Table 9, we found, say, that the fraction of the variance of inflation twelve-periods-ahead explained by:

- the demand shock (`unrate`) is 30.24 per cent;
- the cost-push shock (`inf`) is 68.28 per cent; and
- the monetary policy shock (`ffr`) is a negligible 0.93 per cent

You may ask the – not unreasonable – question of why we might be interested in a theoretical decomposition of **forecast** errors. The latter convey no information about the implications of the structural shocks in the historical sample and have nothing to do with trying to identify the drivers of the business cycle.

Using the concept of a historical decomposition, we can estimate the individual contributions of each structural shock to the movements in the output gap, inflation and the short-term interest rate over the sample period. It was first developed by Sims (1980a), although the first paper based upon it would seem to be Burbidge and Harrison (1985). The historical decompositions of each variable into the estimated structural shocks are calculated as follows:

- the SVAR(3) model is written in companion form (i.e., as a VAR(1) model):

$$y_t = c + Ay_t + u_t \tag{12}$$

- using backward substitution and the Wold decomposition, the model variables at each point in time (y_T) can be represented as a function of initial values (y_0) plus the accumulated sum of all the structural shocks of the model:

$$y_T = A^T y_0 + \sum_{k=1}^{T} A^{T-k} u_k \tag{13}$$

The historical decomposition is a much more useful decomposition than the FEVD, since it shows what is driving the variables in y_t over time.

To carry out a historical decomposition in EViews on the two-variable VAR(6) we have just estimated, we can use the **Historical Decomposition** add-in for EViews 7.2, which can be downloaded for free from:

http://www.eviews.com/cgi/ai_download.cgi?ID=hdecomp.aipz

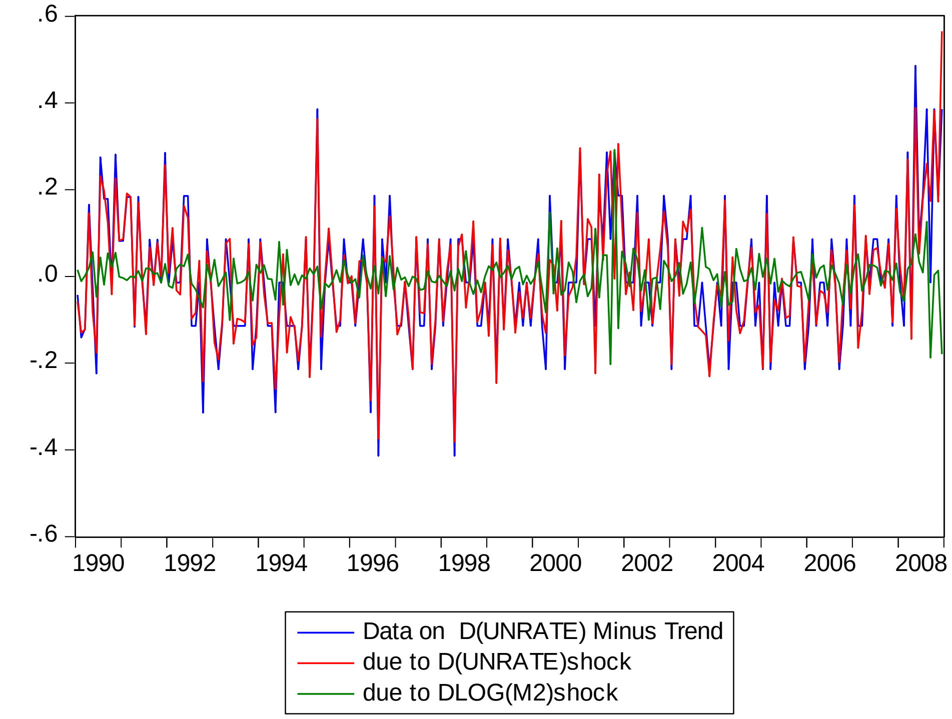
To calculate the historical decomposition of a reduced-form VAR or an identified SVAR in EViews, go to **Proc**, **Add-ins** in the VAR window, i.e., we must have estimated a VAR first.[19] We can use the bivariate VAR(3) on `d(unrate)` and `dlog(m2)` that we have just estimated for this purpose. Upon selecting the **Historical Decomposition** option, EViews will start to carry out the calculations.[20] Once it has finished, it will open a new graph, which shows the historical decompositions, one for each of the variables in the VAR (reproduced in Figure 8).

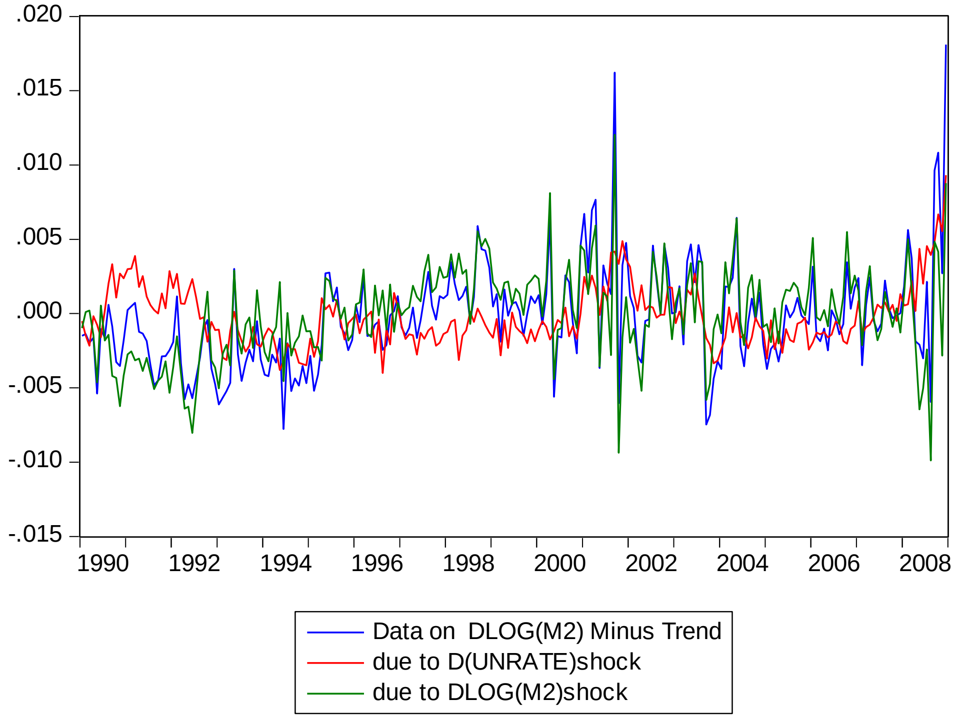
Q13. Calculate the historical decomposition of the change in the unemployment rate as well as the monthly rate of money growth in terms of the two shocks.

[19] While that the VAR can have a constant, other exogenous variables are at the moment not supported. A historical decomposition is also not available for vector error-correction models.

[20] EViews actually calls some Matlab code to do the calculations. This requires not only that Matlab needs to be installed on the computer, but also that EViews can communicate with Matlab.

Figure 8: Historical decomposition





Each graph plots the de-trended data series and the contribution to that series due to each shock. For example, the top graph plots the de-trended change in the unemployment rate (the blue line), the change in the unemployment rate due to its own shock (the red line) and the change in the unemployment rate due to money-supply growth (the green line). Remember that the first shock was labelled a demand shock and the second shock a supply shock.

The underlying numbers are saved in different matrices:

- the matrix `baseout` contains the estimated trend for each endogenous variable in the order they enter the VAR(6); and
- the matrix `histout` contains the contribution of each shock

For each series, there is a base forecast plus the total impact of structural shocks. Hence, the matrix dimension is equal to $n(n + 1)$. In our case, there are two variables (`d(unrate)`) and `dlog(m2)`), so `histmat` will be a (2×3) matrix. Since `d(unrate)` is the first variable, columns 1, 2 and 3 will correspond to the base forecast for `d(unrate)`, historical impact of the first shock and the historical impact of the second shock, respectively. As above, the decompositions are arranged by the shock. The first two columns are the contributions of the demand shock to the two endogenous variables, while the next two columns are the contributions of the supply shock to the two endogenous variables.

Note that these matrices are overwritten every time a new VAR model with sign restrictions is estimated.

6.6 Imposing sign restrictions

Scepticism toward traditional identifying assumptions based on either short- or long-run restrictions has made an alternative class of SVAR models more popular. Starting with Faust (1998), structural shocks in these SVAR models are identified by restricting the sign of the responses of selected model variables to structural shocks. In applied work, Uhlig (2005) has shown that sign-identified models may produce substantially different results from conventionally identified SVAR models. To operationalise the method, identification in sign-identified models requires that each identified shock is associated with a **unique** sign pattern.

Q14. For the original VAR(6) model with the four variables unemployment (`unrate`), the year-on-year growth rate in the nominal M2 money supply (`dm2`), the annual inflation rate (`inf`) and the federal funds target rate (`ffr`), compute the impulse responses using the sign restriction scheme in Table 14. The sample period is `1990M01` to `2008M12`.

Table 14: Sign restriction schemes for four-variable VAR(6)

	Unemployment	CPI	Money	Interest rate
NI	X	X	X	X
NI	X	X	X	X
MD	+	–	+	+
MS	+	–	–	+

In the above Table, X means no restriction, + denotes a restriction that the contemporaneous response is restricted to be positive and – denotes a negative restriction. Furthermore, MS denotes a money-supply shock, MD denotes a money-demand shock and NI means not identified.

To estimate a VAR where the shocks are identified using contemporaneous sign restrictions, we can use the **VAR with sign restrictions** add-in for EViews 7.2.[21]

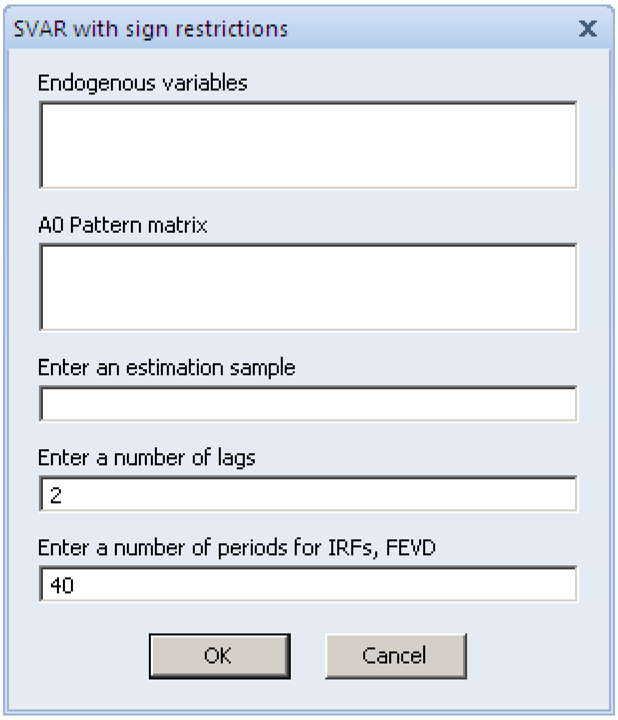
[21] Note that this add-in requires Matlab to be installed on your computer. It also requires that EViews can communicate with Matlab.

Answer. To impose the restrictions, we need to create and name a matrix specifying the sign restrictions. For this example, the EViews matrix looks as follows:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \tag{14}$$

In other words, a positive restriction is specified as 1, a negative restriction as -1 and no restriction as 0. I have saved this matrix as `matrix_sign` in **svar_us.wf1**.

To estimate a VAR where the shocks are identified using contemporaneous sign restrictions, we can use the **VAR with sign restrictions** add-in for EViews 7.2.[22] Go to **Add-ins** and choose **VAR with sign restrictions**. Upon opening the EViews VAR with sign restrictions add-in from the **Add-ins** menu, a dialog box will appear:



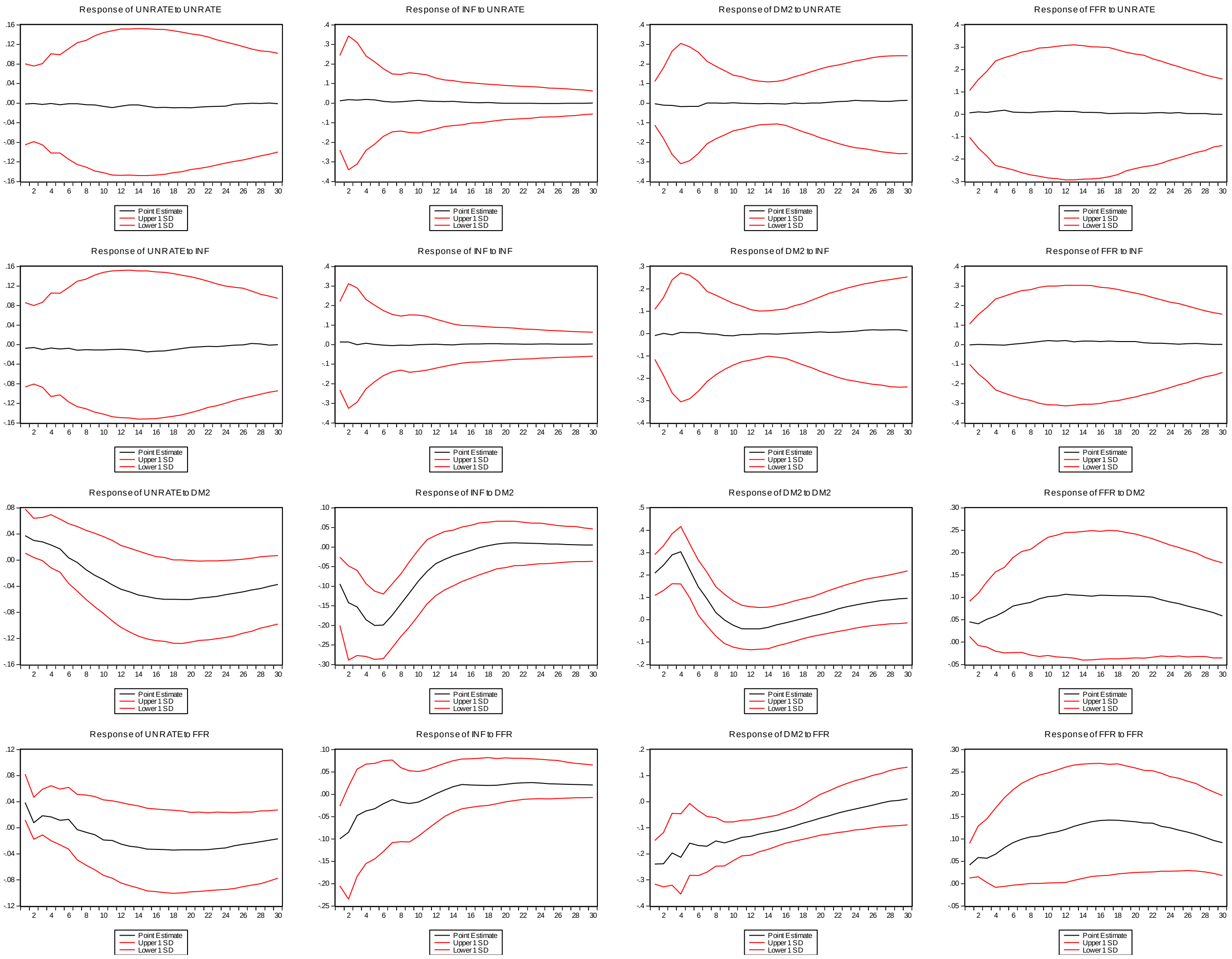
Under **Endogenous Variables**, we need to type the names of the endogenous VAR variables in the order they enter the model, i.e., in a way consistent with the sign restrictions to be imposed. For example, the matrix of sign restrictions created above implies that the variables need to be entered in the order: `unrate inf dm2 ffr`. Under **A0 Pattern matrix**, we need to enter the name of the matrix containing the sign restrictions, i.e., `matrix_sign`. The remaining three boxes (**Enter an estimation sample**, **Enter a number of lags** and **Enter a number of periods for IRFs, FEVD**) are pretty self-explanatory. Note that the estimation sample is still `1990M01 2008M12`, that we have a model with six lags and that we want to plot IRF's over 30 periods.

[22] Note that this add-in also requires Matlab to be installed on your computer. It also requires that EViews can communicate with Matlab.

Clicking on **OK** calls the underlying Matlab code and estimates the VAR model. If estimation proceeds without error, the VAR with sign restrictions add-in produces three graphs: impulse responses, variance decompositions and the historical decomposition.

The first graph, reproduced as Figure 9 below, shows the impulse responses to all shocks in the VAR.

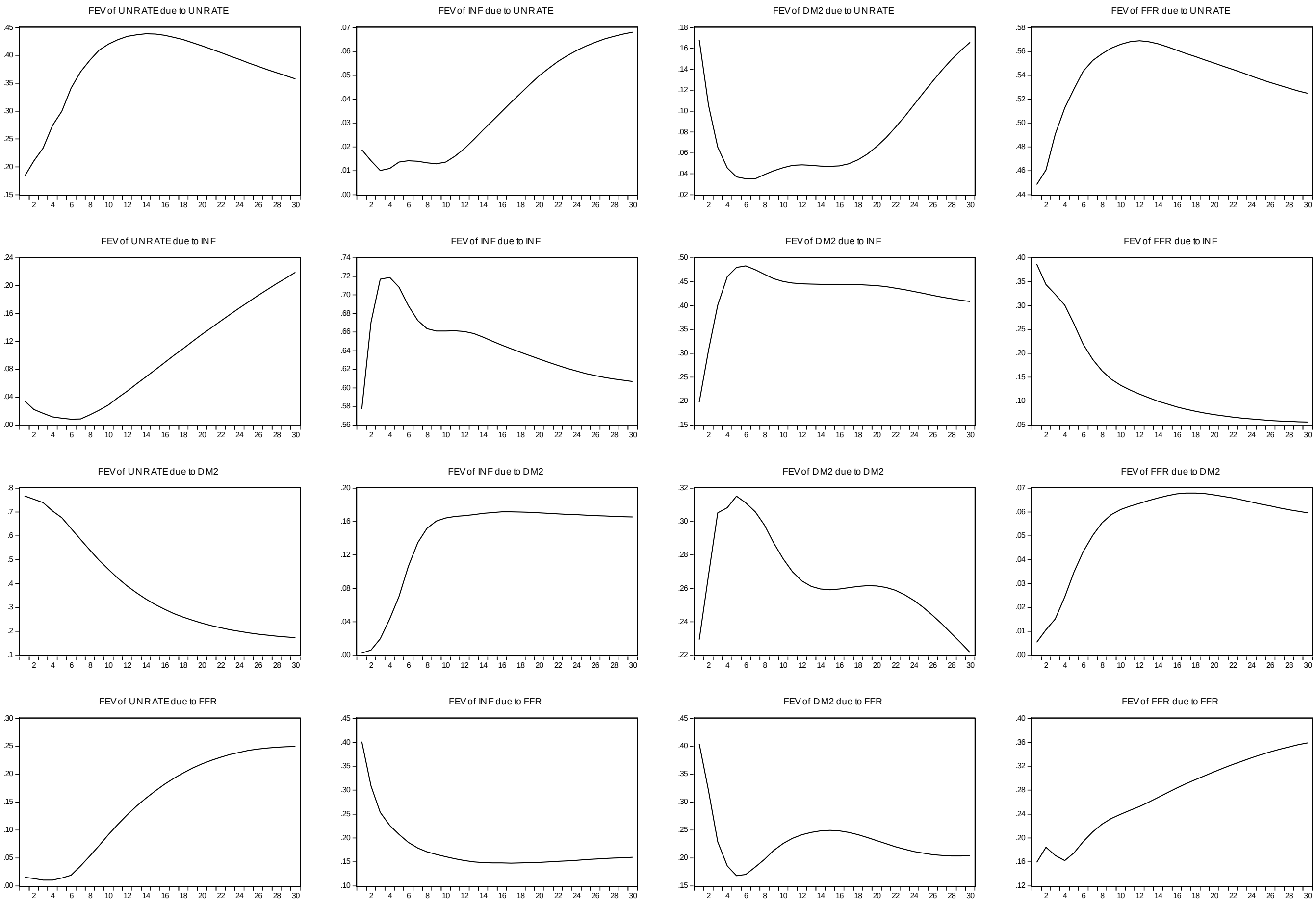
Figure 9: Impulse response functions of `unrate`, `inf`, `dm2` and `ffr` to a one standard-deviation shock in `ffr` (identification by sign restrictions)



In this example, the first two rows show no impulse responses, as we have not imposed any sign restrictions. In other words, we cannot say anything about the response of the other variables in the VAR to structural shocks in `unrate` and `inf`. In contrast, the third row shows the responses to a money-demand shock and the fourth row the responses to a money-supply shock. These impulse response functions are saved in the matrix `mrespout`, where the responses are arranged by the shock. Thus the first four columns are the responses to `unrate`, `inf`, `dm2` and `ffr` to the first (non-identified) shock, the next four columns to the second (non-identified) shock, the following four to the money-demand shock and the final four to the money-supply shock. The matrix `lrespout` contains the lower bound of the IRF confidence interval arranged in the same way, while the matrix `urespout` contains the upper bound.

Figure 10 shows the forecast error variance decomposition, where each row depicts the contribution of each shock.

Figure 10: Forecast error variance decompositions of `unrate`, `inf`, `dm2` and `ffr` to a one standard-deviation shock in `ffr` (identification by sign restrictions)

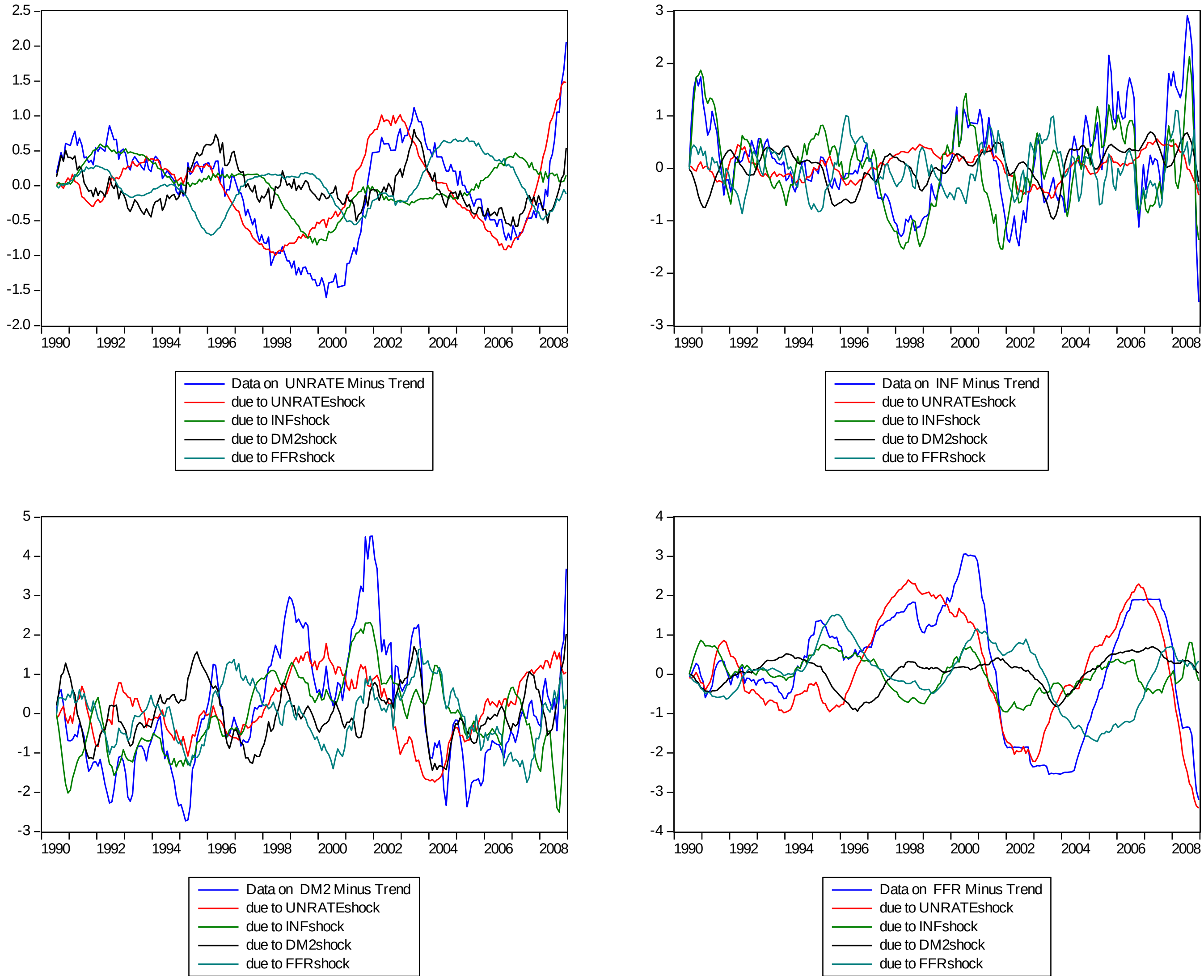


The first row shows the contribution of the shock to `unrate`, the second row shows the contribution of the `inf` shock, the third row shows the contribution of the money-demand shock and the final row shows the contribution of the money-supply shock. Note that the underlying numbers are saved in the matrix `fevdout`, which is arranged in the same manner as the impulse responses.

The third and final graph that is produced by the VAR with sign restrictions add-in shows the historical decomposition, and is reproduced below as Figure 11.

ole.rummel@bankofengland.co.uk

Figure 11: Historical decomposition of `unrate`, `inf`, `dm2` and `ffr` to a one standard-deviation shock in `ffr` (identification by sign restrictions)



Each graph plots the de-trended data series and the contribution to that series due to each shock. For example, the top left-hand graph plots the de-trended unemployment rate (the blue line), the unemployment rate due to the unemployment shock (the red line), the unemployment rate due to the inflation shock (the bright green line), the unemployment rate due to the money-demand shock (the black line) and the unemployment rate due to the money-supply shock (the darker green line).

The underlying numbers are saved in two different matrices:

- the matrix `baseout` contains the estimated trend for each endogenous variable in the order they enter the VAR(6); and
- the matrix `histout` contains the contribution of each shock

As above, these decompositions are arranged by the shock. The first four columns are the contributions of the first (non-identified) shock to the four endogenous variables. The next four columns are the contributions of the second (non-identified) shock to the four endogenous variables, the following four columns are the contributions of the money-demand shock to the four endogenous variables and the final four columns are the contributions of the money-supply shock to the four endogenous variables.

The matrix `betam` contains the point estimates of the VAR coefficients in one (n×(n×p + 1)) vector, where n is the number of endogenous variables and p is the number of lags. The matrix `sigmam` contains the point estimate of the VAR covariance matrix. The matrix `a0newm` contains the point estimate of the estimated A_0 (the structural decomposition) matrix.

Note that these matrices are overwritten every time a new VAR model with sign restrictions is estimated.

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