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ECONOMIC MODELLING AND FORECASTING

Subset VAR, SVAR and VECM modelling

by

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Contents[1]

[1] Sections denoted by an asterisk (*) can be skipped without loss of continuity.

1 Introduction

Vector autoregession (VAR) and structural vector autoregression (SVAR) models have emerged as a dominant tool for the empirical analysis of macroeconomic and financial time series. But, as is well known, both VAR and SVAR analyses suffer from the large number of parameters that have to be estimated and the resulting estimation uncertainty associated with the model's output, such as impulse response functions and forecast error variance decompositions. In other words, as a result of the large number of model parameters in the SVAR, the structural equations of the SVAR are not only imprecisely estimated, but also hard to interpret.

Such considerations point to the need for reductions of the SVAR system. In fact, the application of exclusion restrictions upon the coefficients contained in each structural equation may allow for a better interpretation of the overall system. But the traditional specification of (S)VARs does not allow the number of variables to differ across equations.

The aim of this exercise is to estimate an illustrative small VAR model derived from a dynamic structural general equilibrium (DSGE) model; identify structural shocks (such as a monetary policy shock) by imposing appropriate short-run restrictions using EViews; and assess the results using impulse response functions (IRFs), forecast error variance decompositions (FEVDs) and historical decompositions.

We will start with a small-scale three-equation illustration of the required estimation approach before moving on to one of the main applications of the subset approach to VAR/SVAR modelling, namely block exogeneity and the small open economy assumption.[2]

2 Setting up a (monetary-policy) VAR model

A key consideration before any estimation can be attempted is the form the variables must have when they enter the VAR: should they enter in levels, gaps or first differences? The answer is simply 'it depends'. It depends on what the VAR is going to be used for. For forecasting purposes, we must avoid potential spurious regressions that may result in spurious forecasts; for the purpose of identifying shocks (such as monetary policy shocks) we have to be careful about the stability of the VAR (whether it can be inverted to yield a corresponding vector moving-average representation), the reliability of our impulse response functions and the statistical properties of the residuals.

Q1. Before we estimate our model, what should we do to ensure unbiased estimates?

Answer: We can use OLS to estimate the VAR, so if we are interested in using the estimated model for forecasting, say, we need to ensure that all variables are either stationary or cointegrated to avoid the spurious regression problem associated with unit roots. For structural identification, on the other hand, we are interested in consistent coefficient estimates as well as the interrelationships between the variables, so we follow Canova (2007, p. 125) and 'To minimise pre-testing problems, we recommend starting by assuming covariance stationarity and deviate from it only if the data overwhelmingly suggest the opposite'. As demonstrated by Sims *et al.* (1990), consistent estimates of VAR coefficients are obtained even when unit roots are present.

Moreover, as shown by Toda and Yamamoto (1995) and Dolado and Lütkepohl (1996), if all variables in the VAR are either I(0) or I(1) and if a null hypothesis is considered that does not restrict elements of each of the parameter matrices A_i's (i = 1, 2, … , p), the usual tests have their

[2] Note that the highlighted approach will be equally varied for vector error-correction models (VECMs).

standard asymptotic normal distributions. Moreover, if the VAR order $p \geq 2$, the t-ratios have their usual asymptotic standard normal distributions, meaning that they remain suitable statistics for testing the null hypothesis that a (single) coefficient in one of the parameter matrices is zero (while leaving the other parameter matrices unrestricted). This alleviates the spurious regression problem on the use of standard asymptotic normal distributions.

In light of the results in Sims *et al.* (1990), potential non-stationarity in the VAR under investigation should not affect the model selection process. Moreover, maximum likelihood estimation procedures may be applied to a VAR fitted to the levels even if the variables have unit roots; hence, possible cointegration restrictions are ignored. This is frequently done in (S)VAR modelling to avoid imposing too many restrictions, and we follow this approach here.

3 A small macroeconomic model: Cho and Moreno (2006)

3.1 From a DSGE model to a VAR(p) model

In their simplest incarnation, most multivariate economic models can be written either as a structural VAR (SVAR):

$$B_0 y_t = B_1 y_{t-1} + \varepsilon_t \tag{1}$$

or as a structural moving average:

$$y_t = C_0\varepsilon_t + C_1\varepsilon_{t-1} + C_2\varepsilon_{t-2} + \ldots \tag{2}$$

where y_t is a vector of n variables and there are k ($\leq n$) shocks ε_t that drive the system. This will also be true for dynamic stochastic general equilibrium (DSGE) models, whose general linear (or linearised around equilibrium) specification takes the following form (Sims (2002)):

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + c + \Psi\varepsilon_t + \Pi\eta_t \tag{3}$$

where c is a vector of constants, ε_t is a vector of exogenously evolving random disturbances and η_t is a vector of (rational) expectations errors (with $\mathrm{E}_t(\eta_t) = 0$), not given exogenously, but treated as part of the model process.

The solution of the DSGE model in equation (3) can be expressed as a VAR (see, *inter alia*, Sims (2002)):

$$y_t = A_0 + A_1 y_{t-1} + R\varepsilon_t \tag{4}$$

where the matrices A_0, A_1 and R are convolutions of the underlying structural model parameters and ε_t is a vector of exogenously evolving random disturbances. Written as such, the VARs implied by DSGE models embody a large number of (non-linear) restrictions on its parameters.

A nice illustration of a small new Keynesian (NK) macroeconomic model with rational expectations (RE) is provided by Cho and Moreno (2006). They formulate and solve a variant of the canonical three-equation new Keynesian model that implies non-linear cross-equation restrictions on the dynamics of inflation, the output gap and the policy interest rate. Their micro-founded NK model is given by the following three equations:

$$\pi_t = \delta\mathrm{E}_t(\pi_{t+1}) + (1-\delta)\pi_{t-1} + \lambda y_t + \varepsilon_{AS,t} \tag{5}$$

$$y_t = \mu\mathrm{E}_t(y_{t+1}) + (1-\mu)y_{t-1} - \phi(r_t - \mathrm{E}_t(\pi_{t+1})) + \varepsilon_{IS,t} \tag{6}$$

$$r_t = \alpha_{MP} + \rho r_{t-1} + (1-\rho)[\beta \mathrm{E}_t(\pi_{t+1}) + \gamma y_t] + \varepsilon_{MP,t} \tag{7}$$

Equation (5) is a new Keynesian Phillips curve or aggregate supply equation, which describes the short-run inflation dynamics resulting from the wage-setting process between firms and workers. In equation (5), π_t is inflation between time $(t-1)$ and time t and y_t represents the output gap between time $(t-1)$ and t. $\varepsilon_{AS,t}$ is the aggregate structural supply shock, which is assumed to be independently and identically distributed with homoskedastic variance, σ_{AS}^2. It can be interpreted as a cost-push shock that is either a pricing error or a shock that causes real wages to deviate from their equilibrium value. $E_t(\bullet)$ is the rational expectations operator conditional on the information set at time t, which comprises π_t, y_t and r_t (the nominal interest rate at time t) and all the lags of these variables. λ represents the Phillips curve parameter. As can be seen from equation (5), inflation depends not only on expected future inflation but also on lagged inflation with weights δ and $(1-\delta)$ respectively.[3] As Galí and Gertler (1999) and Woodford (2003) make clear, the endogenous persistence of inflation, as embodied in π_{t-1}, arises from the existence of price setters who do not adjust optimally and index their prices with respect to past inflation.

The (dynamic) IS or demand equation (6) is based on the representative agent's intertemporal utility maximisation with external habit persistence, as proposed by Fuhrer (2000). In equation (6), $\varepsilon_{IS,t}$ is the IS or demand shock, assumed to be independently and identically distributed with homoskedastic variance, σ_{IS}^2. Endogenous persistence of the output gap arises from the habit formation specification in the utility function. The forward-looking parameter, μ, depends inversely on the level of habit persistence. The monetary policy channel in the IS equation is captured by the contemporaneous output gap dependence on the *ex ante* real interest rate. Finally, the monetary transmission mechanism depends negatively on the curvature parameter in the utility function, ϕ.

We close the model with the (forward-looking) monetary policy rule formulated by Clarida *et al.* (2000), given by equation (7). α_{MP} is a constant representing the equilibrium or natural rate of interest and $\varepsilon_{MP,t}$ is the monetary policy shock, assumed to be independently and identically distributed with homoskedastic variance σ_{MP}^2. The monetary policy rule exhibits interest-rate smoothing, placing a weight of ρ on last period's interest rate. The central bank reacts to both deviations of output from trend (the output gap) and high expected inflation (possibly above some inflation target). The parameter β measures the long-run response of the central bank to expected inflation, whereas the parameter γ describes its reaction to output-gap fluctuations.

The macroeconomic system given by equations (5) to (7) can be expressed in matrix form as:

$$\begin{pmatrix} 1 & -\lambda & 0 \\ 0 & 1 & \varphi \\ 0 & -(1-\rho)\gamma & 1 \end{pmatrix}\begin{pmatrix} \pi_t \\ y_t \\ r_t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \alpha_{MP} \end{pmatrix} + \begin{pmatrix} \delta & 0 & 0 \\ \varphi & \mu & 0 \\ (1-\rho)\beta & 0 & 0 \end{pmatrix}\begin{pmatrix} E_t(\pi_{t+1}) \\ E_t(y_{t+1}) \\ E_t(r_{t+1}) \end{pmatrix} + \\ + \begin{pmatrix} (1-\delta) & 0 & 0 \\ 0 & (1-\mu) & 0 \\ 0 & 0 & \rho \end{pmatrix}\begin{pmatrix} \pi_{t-1} \\ y_{t-1} \\ r_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{AS,t} \\ \varepsilon_{IS,t} \\ \varepsilon_{MP,t} \end{pmatrix} \tag{8}$$

In more compact notation:

[3] One advantage of this specification is that it captures the inflation persistence that is present in the data.

$$B_{11}X_t = \alpha + A_{11}\mathrm{E}_t(X_{t+1}) + B_{12}X_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim (0, D) \tag{9}$$

where $X_t = (\pi_t, y_t, r_t)'$, B_{11}, A_{11} and B_{12} are the matrices of structural parameters, α is a vector of constants and ε_t is the vector of structural errors with a (3×1) mean vector of zeros and a diagonal structural error variance-covariance matrix D, i.e., $\varepsilon_t \sim (0, D)$.

The non-linear cross-equation restrictions really come to the fore when we multiply through equation (9) by the inverse of B_{11}:

$$\begin{pmatrix}\pi_t\\ y_t\\ r_t\end{pmatrix} = \begin{pmatrix}1 & -\lambda & 0\\ 0 & 1 & \phi\\ 0 & -(1-\rho)\gamma & 1\end{pmatrix}^{-1}\begin{pmatrix}0\\ 0\\ \alpha_{MP}\end{pmatrix} + \begin{pmatrix}1 & -\lambda & 0\\ 0 & 1 & \phi\\ 0 & -(1-\rho)\gamma & 1\end{pmatrix}^{-1}\begin{pmatrix}\delta & 0 & 0\\ \phi & \mu & 0\\ (1-\rho)\beta & 0 & 0\end{pmatrix}\begin{pmatrix}E_t(\pi_{t+1})\\ E_t(y_{t+1})\\ E_t(r_{t+1})\end{pmatrix}$$

$$+\begin{pmatrix}1 & -\lambda & 0\\ 0 & 1 & \phi\\ 0 & -(1-\rho)\gamma & 1\end{pmatrix}^{-1}\begin{pmatrix}(1-\delta) & 0 & 0\\ 0 & (1-\mu) & 0\\ 0 & 0 & \rho\end{pmatrix}\begin{pmatrix}\pi_{t-1}\\ y_{t-1}\\ r_{t-1}\end{pmatrix} + \begin{pmatrix}1 & -\lambda & 0\\ 0 & 1 & \phi\\ 0 & -(1-\rho)\gamma & 1\end{pmatrix}^{-1}\begin{pmatrix}\varepsilon_{AS,t}\\ \varepsilon_{IS,t}\\ \varepsilon_{MP,t}\end{pmatrix}$$

$$= \frac{1}{1+(1-\rho)\gamma\phi}\begin{pmatrix}-\lambda\phi\alpha_{MP}\\ -\phi\alpha_{MP}\\ \alpha_{MP}\end{pmatrix} + \frac{1}{1+(1-\rho)\gamma\phi}\begin{pmatrix}(1+(1-\rho)\gamma\phi)\delta + \lambda\phi - (1-\rho)\beta\lambda\phi & \mu\lambda & 0\\ \phi - (1-\rho)\beta\phi & \mu & 0\\ (1-\rho)\gamma\phi + (1-\rho)\beta & (1-\rho)\gamma\mu & 0\end{pmatrix}\begin{pmatrix}E_t(\pi_{t+1})\\ E_t(y_{t+1})\\ E_t(r_{t+1})\end{pmatrix}$$

$$+\frac{1}{1+(1-\rho)\gamma\phi}\begin{pmatrix}(1+(1-\rho)\gamma\phi)(1-\delta) & (1-\mu)\lambda & -\lambda\phi\rho\\ 0 & (1-\mu) & -\phi\rho\\ 0 & (1-\rho)(1-\mu)\gamma & \rho\end{pmatrix}\begin{pmatrix}\pi_{t-1}\\ y_{t-1}\\ r_{t-1}\end{pmatrix}$$

$$+\frac{1}{1+(1-\rho)\gamma\phi}\begin{pmatrix}1+(1-\rho)\gamma\phi & \lambda & -\lambda\phi\\ 0 & 1 & -\phi\\ 0 & (1-\rho)\gamma & 1\end{pmatrix}\begin{pmatrix}\varepsilon_{AS,t}\\ \varepsilon_{IS,t}\\ \varepsilon_{MP,t}\end{pmatrix}$$

Note that this is not the equivalent VAR representation of the underlying DSGE model, as we still have to account for the rational expectation terms, $\mathrm{E}_t(\pi_{t+1})$, $\mathrm{E}_t(y_{t+1})$ and $\mathrm{E}_t(r_{t+1})$. Using the solution method outlined in Cho and Moreno (2002), the implied reduced form of the structural model is simply a VAR(1) with highly non-linear parameter restrictions (which can be compared to equation (4) above):

$$X_{t+1} = c + \Omega X_t + \Gamma\varepsilon_{t+1} \tag{10}$$

Two things are worth noting at this point. One, while the linearised RE model solution given by equation (9) above provides a natural **structural interpretation** of the macro dynamics, the **empirical fit** to the data, with one lag only, cannot be as accurate as in large VAR systems that allow for more lags.[4] This is an inherent feature of DSGE models and arises from the linearisation around steady-state, and there is little we can do about this fact. As pointed out by Clarida *et al.* (1999), it is a daunting task to justify structural macroeconomic models of the NK type which include more than one lag. Two, there is a rough correspondence between the DSGE model and

[4] As we will see below, we can use statistical selection methods such as the information criteria to choose the appropriate (higher-order) lag length in non-structural VAR models of monetary policy analysis.

the lag order of the equivalent VAR solution. It turns out that the lag order *p* of the VAR(*p*) model will depend on the statistical properties of the structural innovations, ε_t. When all three structural shock processes are stationary, which is the assumption underlying Cho and Moreno, the equivalent VAR will be a VAR(1) model with one lag. In other words, the implied reduced-form of Cho and Moreno's structural model will be a VAR(1) with highly non-linear parameter restrictions. Using full-information maximum-likelihood (FIML) estimation techniques, Cho and Moreno find the following empirical result for their VAR(1) model:

$$\begin{pmatrix} \pi_t \\ y_t \\ r_t \end{pmatrix} = \begin{pmatrix} 0.670 \\ 0.258 \\ 0.579 \end{pmatrix} + \begin{pmatrix} 0.782* & 0.056 & -0.011 \\ -0.002 & 0.961* & -0.031 \\ 0.154* & 0.114* & 0.838* \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ y_{t-1} \\ r_{t-1} \end{pmatrix} + \begin{pmatrix} 1.772* & 0.106 & -0.013 \\ -0.004 & 1.870* & -0.037 \\ 0.350 & 0.221* & 0.991 \end{pmatrix} \begin{pmatrix} \varepsilon_{AS,t} \\ \varepsilon_{IS,t} \\ \varepsilon_{MP,t} \end{pmatrix} \tag{11}$$

But in the literature, structural errors are often assumed to be serially correlated to fit the data. If the errors $\varepsilon_{AS,t}$, $\varepsilon_{IS,t}$ and $\varepsilon_{MP,t}$ follow a VAR(1), i.e., they are all first-order serially correlated, which is the usual assumption made in the literature, the resulting VAR will be a VAR(2).

3.2 Estimating the Cho and Moreno (2006) model

To open the EViews workfile from within EViews, choose **File**, **Open, EViews Workfile…**, select **chomoreno.wf1** from the appropriate folder and click on **Open**. Alternatively, you can double-click on the workfile icon outside of EViews, which will open EViews automatically. The initial dataset includes the following three variables, from which the sample mean has been removed, i.e., they have been mean-corrected:

- the GDP gap, defined as log GDP after linear detrending (`gap`);
- the inflation rate based on the GDP deflator (`infl`); and
- the federal funds target rate (`ff`)

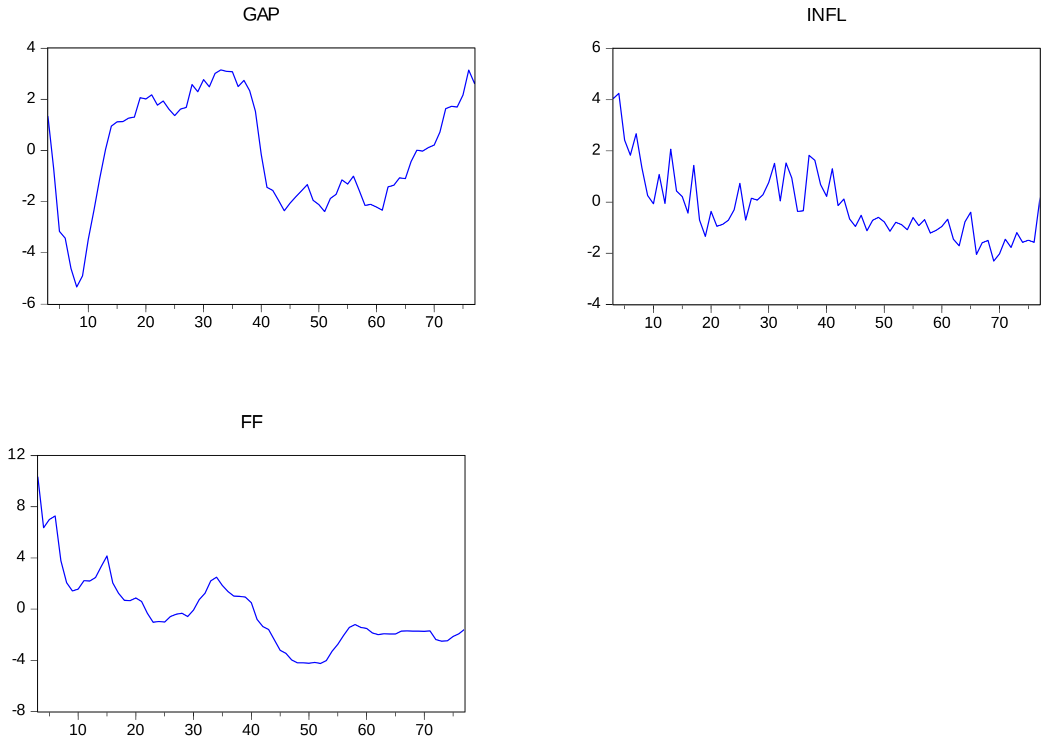
The data are quarterly and span the period from 1981 Q1 (1981Q1 in EViews notation) to 2000 Q1 (2000Q1). Unless we are confident in assuming that the underlying monetary policy shocks are robust to different monetary policy regimes (money-supply targeting, exchange-rate targeting, inflation targeting, etc.) and changes in regimes over time, it is important to estimate parameters in structural vector autoregressions (SVARs) on a **single** policy regime. Any regime shift therefore requires a different parameterisation of the SVAR model. This important *caveat* may explain some of the counterintuitive results we will encounter in the following exercises, in which VARs are estimated and SVARs identified over long time periods.[5]

Whenever we begin working with a new data set, it is always a good idea to take some time to simply examine the data, so the first thing we will do is to plot the data to make sure that it looks fine. This will help ensure that there were no mistakes in the data itself or in the process of

[5] For example, many structural VAR studies of US monetary policy leave out the disinflationary period from 1979 to 1984, which constitutes a different monetary policy regime.

reading in the data. It also provides us with a chance to observe the general (time-series) behaviour of the series we will be working with. A plot of our data is shown in Figure 1 below.

Figure 1: US output gap (`gap`), inflation (`infl`) and federal funds rate (`ff`)



Q2. Estimate an unrestricted VAR(2). We start our estimation with `gap`, `infl` and `ff`, including also a constant. The role of the constant at this point may be contentious, as all the data series are mean-corrected.

Answer: Use **Quick**, **Estimate VAR…**, enter `gap`, `infl` and `ff` in that order into the **Endogenous Variables** box and leave EView's default setting of `1 2` for the lag interval. The sample period for estimation should be 1981 Q1 to 2000 Q1. An equivalent way of getting EViews to do the estimation is to type the following command in the command window:

```
var var_2lags.ls 1 2 gap infl ff
```

This command specifies a VAR with the name **var_2lags** with an initially arbitrary lag length of two. As you can see from Table 1 below, even estimating a small three-variable VAR(2) generates a lot of output. Each column in the table corresponds to the equation for one endogenous variable in the VAR. For each right-hand side variable, EViews reports the coefficient point estimate, the estimated coefficient standard error (in round brackets) and the *t*-statistic (in square brackets).

Table 1: Estimation results for the VAR(2) model in `gap`, `infl` and `ff` (constant, 1981 Q1 – 2000 Q1)

Vector Autoregression Estimates
Sample (adjusted): 1981Q3 2000Q1
Included observations: 75 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	GAP	INFL	FF
GAP(-1)	1.216215	0.248145	0.190569
	(0.12259)	(0.15334)	(0.14732)
	[9.92075]	[1.61823]	[1.29358]
GAP(-2)	-0.295935	-0.197354	-0.120531
	(0.11954)	(0.14952)	(0.14365)
	[-2.47565]	[-1.31989]	[-0.83907]
INFL(-1)	-0.058829	0.347231	0.124601
	(0.09654)	(0.12075)	(0.11601)
	[-0.60939]	[2.87557]	[1.07407]
INFL(-2)	-0.041853	0.241968	0.177445
	(0.09345)	(0.11689)	(0.11230)
	[-0.44785]	[2.06996]	[1.58007]
FF(-1)	0.193684	0.050672	0.916828
	(0.10324)	(0.12914)	(0.12406)
	[1.87606]	[0.39239]	[7.39008]
FF(-2)	-0.187740	0.059209	-0.151764
	(0.09187)	(0.11492)	(0.11040)
	[-2.04345]	[0.51522]	[-1.37462]
C	0.038279	-0.120669	-0.150377
	(0.07069)	(0.08842)	(0.08494)
	[0.54153]	[-1.36476]	[-1.77032]
R-squared	0.931354	0.715724	0.942066
Adj. R-squared	0.925297	0.690641	0.936954
Sum sq. resids	23.82587	37.27764	34.40574
S.E. equation	0.591929	0.740406	0.711313
F-statistic	153.7659	28.53404	184.2900
Log likelihood	-63.41854	-80.20435	-77.19796
Akaike AIC	1.877828	2.325449	2.245279
Schwarz SC	2.094127	2.541748	2.461578
Mean dependent	-0.046347	-0.154104	-0.265200
S.D. dependent	2.165719	1.331185	2.832899
Determinant resid covariance (dof adj.)		0.077591	
Determinant resid covariance		0.057830	
Log likelihood		-212.3769	
Akaike information criterion		6.223385	
Schwarz criterion		6.872281	

As anticipated, the constant does not enter significantly into any of the three equations (remember that the data has been mean-corrected). We can therefore re-estimate the model by ignoring the constant. When we re-estimate the VAR(2) model, leave the **Exogenous Variables** box blank, i.e., remove the `c` for the constant from the box. Results for the re-estimated model are given in Table 2.

Table 2: Estimation results for the VAR(2) model in `gap`, `infl` and `ff` (no constant, 1981 Q1 – 2000 Q1)

Vector Autoregression Estimates
Sample (adjusted): 1981Q3 2000Q1
Included observations: 75 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	GAP	INFL	FF
GAP(-1)	1.221168	0.232532	0.171111
	(0.12162)	(0.15387)	(0.14916)
	[10.0405]	[1.51123]	[1.14715]
GAP(-2)	-0.300272	-0.183681	-0.103493
	(0.11866)	(0.15012)	(0.14552)
	[-2.53058]	[-1.22359]	[-0.71117]
INFL(-1)	-0.066145	0.370296	0.153343
	(0.09510)	(0.12031)	(0.11663)
	[-0.69556]	[3.07788]	[1.31480]
INFL(-2)	-0.035161	0.220873	0.151157
	(0.09216)	(0.11659)	(0.11302)
	[-0.38154]	[1.89443]	[1.33739]
FF(-1)	0.182381	0.086302	0.961229
	(0.10059)	(0.12726)	(0.12336)
	[1.81314]	[0.67817]	[7.79177]
FF(-2)	-0.177073	0.025582	-0.193669
	(0.08928)	(0.11295)	(0.10949)
	[-1.98341]	[0.22650]	[-1.76881]
R-squared	0.931058	0.707938	0.939395
Adj. R-squared	0.926063	0.686774	0.935004
Sum sq. resids	23.92862	38.29870	35.99145
S.E. equation	0.588890	0.745019	0.722229
F-statistic	186.3693	33.45017	213.9056
Log likelihood	-63.57991	-81.21769	-78.88764
Akaike AIC	1.855464	2.325805	2.263670
Schwarz SC	2.040863	2.511204	2.449069
Mean dependent	-0.046347	-0.154104	-0.265200
S.D. dependent	2.165719	1.331185	2.832899

Determinant resid covariance (dof adj.)	0.081383
Determinant resid covariance	0.063372
Log likelihood	-215.8085
Akaike information criterion	6.234894
Schwarz criterion	6.791091

We note in passing some statistical evidence that a number of coefficients in the B_j (j = 1, 2) matrices are not statistically significant, i.e., equal to zero. For example, `gap(-2)` is significant in the VAR equation for `gap`, but not in the VAR equations for either `infl` or `ff`. On the other hand, we find that both the first and the second lag are jointly significant. This test is produced by going to **View**, **Lag Structure** and **Lag Exclusion Tests**. Results for this test are given in Table 3.

Table 3: VAR(2) lag exclusion tests

VAR Lag Exclusion Wald Tests
Sample: 1981Q1 2000Q1
Included observations: 75

Chi-squared test statistics for lag exclusion:
Numbers in [ ] are p-values

	GAP	INFL	FF	Joint
Lag 1	147.3858	12.57832	84.29691	238.5151
	[0.000000]	[0.005643]	[0.000000]	[0.000000]
Lag 2	14.95736	4.183361	6.153020	24.82308
	[0.001853]	[0.242333]	[0.104398]	[0.003173]
df	3	3	3	9

The null hypothesis of the tests displayed in Table 3 is that all of the variables constituting the first and the second lag are jointly insignificant. The final column shows the associated $\chi^2(9)$-statistic as well as its probability value. As we can see from Table 3, the *p*-values indicating the joint significance of the entire lag are all below 0.05, indicating that we can reject the null hypothesis that either the first or the second lag can be excluded.

Q3. We have selected an arbitrary lag length of 2, but is that appropriate?

The choice of *p*, the lag length of the VAR, is motivated by three different considerations:

- rules-of-thumb;
- a lag length suggested by **theoretical models**; or
- **statistical criteria** that trade off fit against the number of parameters fitted

Overall, **rules-of-thumb** provide (convenient) starting values for *p* at best. Some widely-used rules-of-thumb depend on the frequency of your data: if you have annual data you could start with 2-3 lags; if you have quarterly data you could start with, say, 8-10 lags; and if you have monthly data with 18-24 lags. For the problem at hand, we could also rely on **theory** to provide some guidance. We normally make the assumption that the shocks in a new Keynesian model individually follow AR(1) processes (or a VAR(1) collectively). If this should be the case, the solution to the model is a VAR(2). In fact, it turns out that the solution to a DSGE model is generally a VAR(2). This also means that if the shocks are VAR(0), as in the Cho and Moreno (2006) model considered here, the solution collapses to a VAR(1) model. This suggests that we may only need to estimate a VAR(1) model.[6]

In general, however, we employ **statistical** model selection criteria to help us choose the appropriate lag length of the VAR.

[6] This seems to be the most appropriate approach if we wanted to compare an estimated DSGE model to an estimated VAR model.

Answer: Too short a lag length may result in inconsistent estimates, while too many lags can result in imprecise estimates in small and moderate samples. As such, adding more lags improves the fit but reduces the degrees of freedom and increases the danger of over-fitting. An objective way to decide between these competing objectives is to maximise some weighted measure of these two parameters. This is how the Akaike information criterion (**AIC**), the Schwarz or Bayesian criterion (**SC**) and the Hannan-Quinn criterion (**HQ**) work. These three statistics are measures of the trade-off of improved fit against loss of degrees of freedom, so that the best lag length should minimise all of these three statistics.[7]

An alternative to the information criterion is to systematically test for the significance of each lag using a likelihood-ratio test (Lütkepohl (1991), Section 4.3). This is the approach favoured by Sims (1980a), who also suggested a modification to the likelihood-ratio (LR) test to take into account small-sample bias. We should follow his recommendation in practice – EViews does as well! Since an unrestricted VAR of lag length p nests the same restricted VAR of lag length (p – 1), the log-likelihood difference multiplied by the number of observations less the number of regressors in the VAR should be distributed as a χ^2-distribution with degrees of freedom equal to the number of restrictions **in the system**, s, i.e., $\chi^2(s)$. In other words:

$$LR = (T - m)\ \{\log|\Omega_{p-1}| - \log|\Omega_p|\} \sim \chi^2(s) \qquad (12)$$

where T is the number of observations, m is the number of parameters estimated per equation (under the alternative) and $\log|\Omega_j|$ is the logarithm of the determinant of the variance-covariance matrix Ω of the VAR with j lags, where $j = 0, 1, 2, \ldots, p$. The adjusted test has the same asymptotic distribution as the standard likelihood-ratio test that does not include the adjustment for m, but is less likely to reject the null hypothesis in small samples. For each lag length, if there is no improvement in the fit from the inclusion of the last lag then the difference in errors should not be significantly different from white noise. Make sure that you use the same sample period for the restricted and the unrestricted model, i.e., do not use the extra observation that becomes available when you shorten the lag length.[8] For our example, we use a general-to-specific methodology (some authors specify specific-to-general instead, which I would strongly discourage):[9]

(i) we start with a high lag length (say 8 lags for quarterly data);
(ii) for each lag, say p, note the determinant of the residual variance-covariance matrix given in the EViews output and then note the determinant of the residual variance-covariance matrix for a VAR of lag (p – 1);
(iii) take the difference of the logs of the determinants; and
(iv) this difference multiplied by (T – m) should be distributed as a χ^2-distribution with s degrees of freedom.[10]

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[7] Some programs maximise the negative of these measures.
[8] EViews will do this automatically.
[9] An informative reference in strong support of the general-to-specific approach is Lütkepohl (2007).
[10] The degrees of freedom will depend on the number of variables as well as the number of lags in the VAR. The total number of variables in a VAR is given by $n(1 + np) = n + n^2p$, where n is the number of variables and p is the number of lags. In our case of a three-variable VAR(2), estimating the model with four lags rather than five means that we have three less parameters to estimate per equation. In total, we will have nine less parameters in the system – in general, n^2. The number of restrictions, s, in the system would therefore be 3*3 = 9.

It is also very important to note that the **residuals** from the estimated VAR should be well behaved, that is, there should be no problems with autocorrelation and non-normality. Thus, whilst the AIC, SC or HQ may be good starting points for determining the lag-length of the VAR, it is important to check for residual autocorrelation and non-normality. If we find that there is autocorrelation for the chosen lag-length, one ought to increase the lag-length of the VAR until the problem disappears. Similarly, if there are problems with non-normality, a useful trick is to add exogenous variables to the VAR (they may correct the problem), including dummy variables and a time trend.

At the same time, we should note that the specification of a VAR and its statistical adequacy is an issue that has not received much explicit attention in the literature. In most of the applied papers, the lag length is either decided uniformly on an *ad hoc* basis, several different lag lengths are estimated for 'robustness' or the lag length is simply set on the basis of information criteria. Virtually none of the academic papers cited in this exercise undertake any rigorous assessment of the statistical adequacy of the estimated models.

Q4. Test for the appropriate number of lags using the AIC, SC, HQ and the LR information criterion.

Answer: In our VAR window, select **View**, **Lag Structure**, **Lag Length Criteria…**, then enter 8 in the maximum lag specification. The readout should be as in Table 4.

Table 4: VAR lag order selection criteria

Endogenous variables: GAP INFL FF
Exogenous variables:
Sample: 1981Q1 2000Q1
Included observations: 69

Lag	LogL	LR	FPE	AIC	SC	HQ
1	-176.7669	NA	0.043765	5.384548	5.675953	5.500158
2	-150.1501	48.60464*	0.026295*	4.873915*	5.456726*	5.105136*
3	-141.6341	14.81031	0.026751	4.887946	5.762162	5.234777
4	-134.7852	11.31562	0.028649	4.950296	6.115917	5.412737
5	-129.1655	8.796092	0.031930	5.048275	6.505301	5.626326
6	-126.3311	4.189976	0.038789	5.226988	6.975419	5.920649
7	-120.6421	7.915096	0.043677	5.322960	7.362796	6.132231
8	-113.7900	8.937596	0.047952	5.385216	7.716458	6.310098

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

The LR, AIC, SC and HQ information criteria all suggest 2 lags respectively, which is indicated by the asterisk (*) next to the test statistic. This is a rare occurrence in applied work, as most of the time these information criteria frequently indicate different lag lengths, meaning that the 'correct' lag length can depend on the criteria or measure we use. This is typical of these tests and researchers often use the criterion most 'convenient' for their needs. Note that the AIC is inconsistent and generally **over**estimates the true lag order with positive probability; but that both SC and HQ are consistent. The SC criterion is generally more conservative in terms of lag length than the AIC criterion, i.e., it selects a shorter lag than the other criteria. Ivanov and Kilian (2005) show that while the choice of information criterion depends on the frequency of the data and type of model, HQ is typically more appropriate for quarterly and monthly data.

Q5. Does the chosen VAR(2) have appropriate properties? Are the residuals **stationary, normal** and **not autocorrelated**? Is the VAR **stable**?

A useful tip is to start with the VAR with the minimum number of lags according to the information criteria (in this case 2 lags) and check whether there are problems with autocorrelation, normality and stability.

Answer (normality): As we are already using a VAR with two lags, there is no need for us to re-estimate the model. Click on **View**, choose **Residual Tests** and pick **Normality Test…**. For the **normality** test we get the results in Table 5.

Table 5: VAR(2) residual normality tests

VAR Residual Normality Tests
Orthogonalization: Cholesky (Lutkepohl)
Null Hypothesis: residuals are multivariate normal
Sample: 1981Q1 2000Q1
Included observations: 75

Component	Skewness	Chi-sq	df	Prob.
1	-0.369409	1.705785	1	0.1915
2	0.539119	3.633115	1	0.0566
3	0.274537	0.942132	1	0.3317
Joint		6.281033	3	0.0987

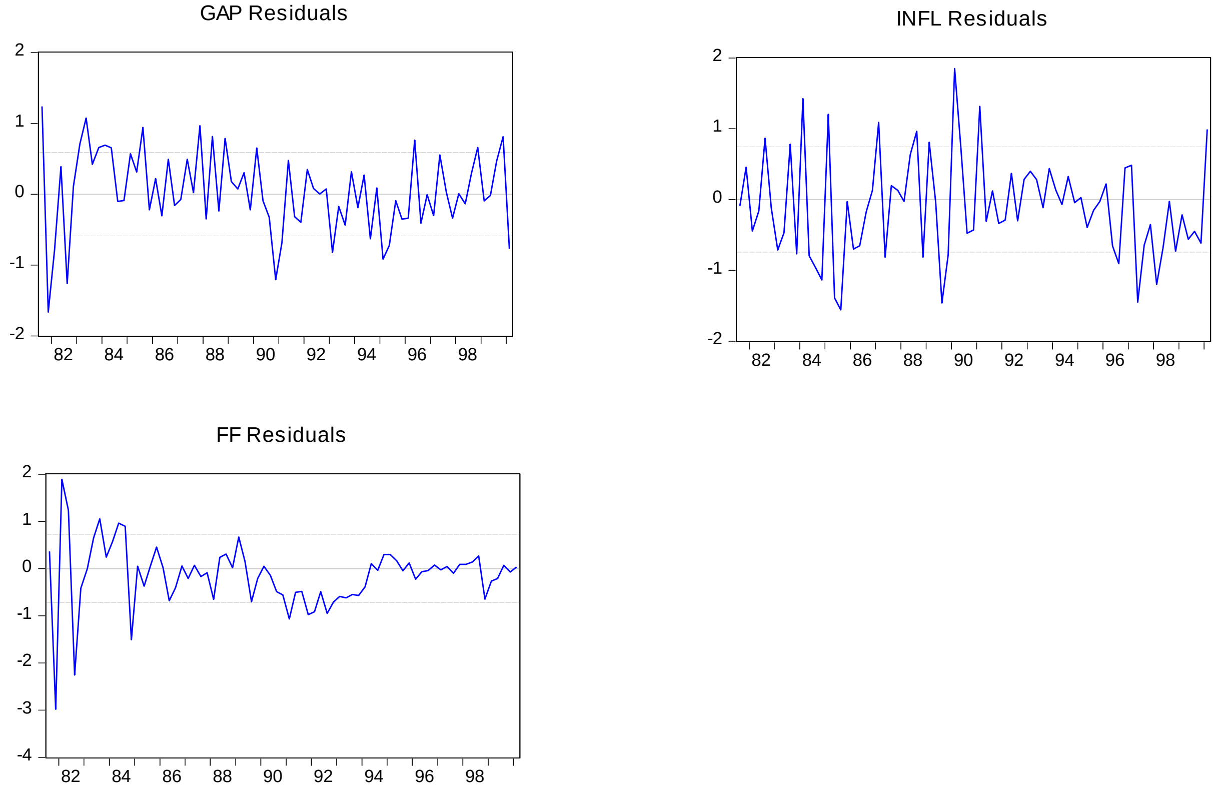
Component	Kurtosis	Chi-sq	df	Prob.
1	3.202966	0.128735	1	0.7197
2	3.387565	0.469395	1	0.4933
3	6.294838	33.92487	1	0.0000
Joint		34.52300	3	0.0000

Component	Jarque-Bera	df	Prob.
1	1.834521	2	0.3996
2	4.102510	2	0.1286
3	34.86701	2	0.0000
Joint	40.80404	6	0.0000

Thus this VAR has problems with non-normality (why?). The reason for this is obvious: we note that it is the third residual series in particular that has problems with (excess) kurtosis, leading to the rejection of joint normality for all three residual series.

The non-normality of the residuals results from a number of very large outliers, as can be seen in Figure 2. Very obvious is the disinflationary policy period from 1979 to 1984 in the `ff` residuals. In addition, inflation (`inf`) shows two very large (negative) residuals towards the end of the sample period. Indeed, looking at the residual diagnostics more closely, we can see that the influence of large outliers in the components, as well as the overall Jarque-Bera tests, is limited to the third residual series. This is, unfortunately, a problem. Although normality is not a necessary condition for the validity of many of the statistical procedures related to VAR and SVAR models, deviations from the normality assumption may nevertheless indicate that improvements to the model are possible.[11]

Figure 2: Estimated residual series from the VAR(2) model with `gap`, `infl` and `ff`



Rather than EViews' default setting of **Cholesky of covariance (Lütkepohl)** as the **Orthogonalisation method**, some authors prefer to use **Square root of correlation (Doornik-Hendry)**. This is because we must choose a factorisation of the residuals for the multivariate normality test, such that residuals are orthogonal to each other. The approach due to Doornik and Hansen (2008) has two advantages over the one in Lütkepohl (1991, p. 155-158). First, Lütkepohl's test uses the inverse of the lower triangular Cholesky factor of the residual covariance matrix, resulting in a test which is not invariant to a re-ordering of the dependent variables. Second, Doornik and Hansen perform a small-sample correction to the transformed residuals before computing their statistics. We should note that the finding of non-normality is robust to the orthogonalisation method, though.

Answer (autocorrelation): For autocorrelation, click on **View**, choose **Residual Tests** and pick the **Autocorrelation LM test…**. The output for the VAR(2) model with twelve lags is given in Table 6.

[11] One way of dealing with this problem would be to introduce dummies to account for some of the larger outliers. An alternative suggestion, following Footnote 5, would be to leave out the first couple of years in the estimation.

Table 6: VAR(2) residual autocorrelation LM tests up to lag order twelve

VAR Residual Serial Correlation LM Tests
Null Hypothesis: no serial correlation at lag order h
Sample: 1981Q1 2000Q1
Included observations: 75

Lags	LM-Stat	Prob
1	13.42597	0.1443
2	18.01048	0.0351
3	13.09121	0.1585
4	12.24270	0.2000
5	5.478050	0.7908
6	5.727160	0.7669
7	4.378134	0.8848
8	3.926748	0.9162
9	15.95829	0.0678
10	4.928498	0.8405
11	15.55670	0.0767
12	7.121232	0.6245

Probs from chi-square with 9 df.

With the exception of second-order autocorrelation, which is significant at the 5 per cent level of significance, there are no other problems with autocorrelation at all remaining lags. To address potential problems of **autocorrelation**, the general remedy is to add further lags to the VAR. But adding a third lag and estimating a VAR(3) does not help in this respect. In fact, it results in residual serial correlation at lags 1 and 2 (this is left as an optional exercise). I have therefore decided to retain the VAR(2) model for further analysis.

Answer (stability): If the VAR is not stable, certain results (such as impulse response standard errors) are not valid. Most importantly, if the VAR is not stable, we will not be able to generate the vector moving-average (VMA) representation from the VAR. In doing this procedure, there will be ($n \times p$) roots overall, where n is the number of endogenous variables (three) and p is the particular lag length (two). It is easy to check for stability in EViews. Go to **View**, **Lag Structure** and click on **AR Roots Table**. You should get the results in Table 7.

Table 7: Roots of the characteristic polynomial of the VAR(2)

Roots of Characteristic Polynomial
Endogenous variables: GAP INFL FF
Exogenous variables:
Lag specification: 1 2

Root	Modulus
0.882492 - 0.078944i	0.886016
0.882492 + 0.078944i	0.886016
0.542793	0.542793
0.290611 - 0.169471i	0.336416
0.290611 + 0.169471i	0.336416
-0.336307	0.336307

No root lies outside the unit circle.
VAR satisfies the stability condition.

The VAR is stable as none of the roots lie outside the unit circle: all the moduli of the roots of the characteristic polynomial are less than one in magnitude. In case one or more roots fall outside the unit circle, adding a time trend (denoted `@trend` in EViews) as an exogenous variable can help the stability of the VAR. Note that adding a time trend will not always correct instability.

We also note that all of the roots of the characteristic polynomial of the VAR are less than 0.9 in magnitude. We will return to this point later on. The main argument is that if the characteristic roots are close to one, it will be doubtful if the analytic (asymptotic) confidence intervals that EViews produces will be accurate. If, on the other hands, one or more of the characteristic roots exceed 0.9 in magnitude, we may want to consider bootstrapping the confidence intervals.

4 VAR identification

When Sims (1980a) first advocated the use of a VAR in economics, it was in response to the prevailing orthodoxy at the time that all economic models should be structural models, i.e., that they should include identifying restrictions. Instead, he argued for the use of an unrestricted VAR with no distinction being made in the model between endogenous and exogenous variables. The aim was to free-up econometric modelling from the constraints applied by economic theory and, in effect, to 'let the data speak.'[12]

We are now ready to attempt identification of the **structural VAR** (SVAR) and, in so doing, identify monetary policy shocks. In EViews, we have an underlying **structural** system of equations of the form:

$$Ay_t = C(L)y_t + Bu_t \tag{13}$$

where the structural shocks u_t are normally distributed, i.e., $u_t \sim N(0, \Sigma)$, where Σ is generally assumed to be a diagonal matrix, usually the identity matrix, such that $u_t \sim N(0, I)$.[13] Unfortunately, we cannot estimate this equation directly due to identification issues. Instead, we estimate an unrestricted or reduced-form VAR of the form:

$$y_t = A^{-1}C(L)y_t + A^{-1}Bu_t$$

$$= H(L)y_t + \varepsilon_t \tag{14}$$

For reasons outlined in the presentation, the matrices A, B and the C_i's ($i = 1, 2, \dots, p$) are not separately observable from the estimated H_i's and the variance-covariance matrix, $E(\varepsilon_t\varepsilon_t') = \Omega$, of the reduced-form shocks, ε_t. So how can we recover equation (13) from equation (14)? The solution is to impose restrictions on our VAR to identify an underlying structure – but what kind of restrictions are these?

Economic theory can sometimes tell us something about the structure of the system we wish to estimate. As economists, we must convert these structural or theoretical assumptions into feasible restrictions on the VAR. Such restrictions can include, for example:

- a causal (recursive) ordering of shock propagation, e.g., the Cholesky decomposition;

[12] But we have already come across at least two necessary restrictions: (i) we need to choose the variables that go into the model and (ii) we have to choose a finite lag length for the VAR.

[13] For reasons having to do with the *AB* model below, I use a different notation from the presentation.

- the fact that nominal variables have no long-run effect on real variables;
- the long-run behaviour of variables, e.g., the real exchange rate is constant in the long run; and
- the fact that we can derive theoretical restrictions on the signs of the impulse responses resulting from particular structural shocks

There are many types of restrictions that can be used to identify a structural VAR. EViews allows you to impose different types of restrictions. One type imposes restrictions on the short-run behaviour of the system, whereas another type imposes restrictions on the long-run. Using the SVARpatterns add-in, EViews even allows you to impose both types of restrictions at the same time, as has been done in Bjørnland and Leitemo (2009). Finally, we can also impose sign restrictions to identify structural VARs.

4.1 Imposing short-run restrictions

To impose short-run restrictions in EViews, we use equation (14):

$$y_t = A^{-1}C(L)y_t + A^{-1}Bu_t$$

We estimate the random stochastic residual $A^{-1}Bu_t$ from the residual ε_t of the estimated VAR. Comparing the residuals from equations (13) and (14), we find that:

$$\varepsilon_t = A^{-1}Bu_t \tag{15}$$

or, equivalently, that:

$$A\varepsilon_t = Bu_t \tag{15′}$$

In requiring that restriction or identifying schemes must be of the form given by (15′) above, EViews follows what is known as the *AB* model, which is extensively described in Amisano and Giannini (1997). By imposing structure on the matrices *A* and *B*, we impose restrictions on the structural VAR in equation (13).

Reformulating equation (15), we have $\varepsilon_t\varepsilon_t' = A^{-1}Bu_tu_t'B'(A^{-1})'$, and, since $E(u_tu_t') = I_n$ (the $(n \times n)$ identity matrix) by assumption, we have:

$$E(\varepsilon_t\varepsilon_t') = E(A^{-1}Bu_tu_t'B'(A^{-1})') = A^{-1}B\, E(u_tu_t')\, B'(A^{-1})' = A^{-1}BB'(A^{-1})' = \Omega \tag{16}$$

But can we identify all the elements in *A* and *B* from Ω? Equation (16) says that for the *n* variables in y_t, the symmetry property of the variance-covariance matrix $E(\varepsilon_t\varepsilon_t') = \Omega$ imposes $n(n + 1)/2$ (identity) restrictions on the $2n^2$ unknown elements in *A* and *B*. Thus, an additional $2n^2 - n(n + 1)/2 = (3n^2 - n)/2$ restrictions must be imposed.

In our case, and using the above *AB* model, we have a VAR with three endogenous variables, requiring (3*3*3 – 3)/2 = 12 restrictions. An example of this specification using the Cholesky decomposition identification scheme is:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ a_{21} & 1 & 0 \\ a_{31} & a_{32} & 1 \end{pmatrix}, B = \begin{pmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{pmatrix} \quad (17)$$

where the *A* matrix by dint of being lower triangular has a recursive structure. Counting restrictions in the *A* and *B* matrices above, we have nine zero restrictions (three in matrix *A* and six in matrix *B*) as well as another three normalisation restrictions on the diagonal of matrix *A*, giving us the required total of twelve restrictions. In EViews, these restrictions can be imposed in either matrix form or in text form. Since imposing restrictions in matrix form is relatively straightforward, we will start off by illustrating the text form.[14]

Q6. Impose the Cholesky decomposition, which assumes that shocks or innovations are propagated in the order of `gap`, `infl` and `ff`. As we will discuss below, the Cholesky decomposition can be interpreted as a recursive contemporaneous structural model.[15]

Answer: To impose the restriction above in **text format**, we select **Proc** and **Estimate Structural Factorisation…** from the VAR window menu. In the SVAR options dialog, select **Text** (or **Matrix** as appropriate). Following equation (15), each endogenous variable has an associated variable number, in our example this is:

- `@e1` for the `gap` residuals;
- `@e2` for the `inf` residuals; and
- `@e3` for the `ff` residuals

The identifying restrictions are imposed in terms of the ε's, which are the residuals from the reduced-form VAR estimates, and the *u*'s, which are the structural, fundamental or 'primitive' random (stochastic) errors in the structural system. Enter the following in the text box (it is easiest to simply copy the suggested short-run factorisation example from the top of the SVAR options box into the white **Identifying Restrictions** box at the bottom):

```
@e1 = c(1)*@u1
@e2 = c(2)*@e1 + c(3)*@u2
@e3 = c(4)*@e1 + c(5)*@e2 + c(6)*@u3
```

The way to interpret these restrictions is that they represent the entries in the $A^{-1}B$ matrix linking ε_t and u_t *via* equation (17), i.e., $\varepsilon_t = A^{-1}Bu_t$. The following should hopefully make this a bit clearer.

We make use of the fact that the inverse of a lower (upper) triangular matrix is also a lower (upper) triangular matrix. We can take a closer look at the underlying matrix algebra that results in the set of EViews restrictions. Writing out equation (17) and the identification restrictions in full matrix form, we have:

$$A\varepsilon_t = Bu_t$$

[14] Restrictions using text form are also more flexible, as we can restrict values to be equal.
[15] We should note, however, that most economic theories do not imply recursive contemporaneous systems.

$$\begin{bmatrix} 1 & 0 & 0 \\ a_{21} & 1 & 0 \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix} = \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ a_{21} & 1 & 0 \\ a_{31} & a_{32} & 1 \end{bmatrix}^{-1} \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -a_{21} & 1 & 0 \\ a_{21}a_{32} - a_{31} & -a_{32} & 1 \end{bmatrix} \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix}$$

$$= \begin{bmatrix} b_{11} & 0 & 0 \\ -a_{21}b_{11} & b_{22} & 0 \\ (a_{21}a_{32} - a_{31})b_{11} & -a_{32}b_{22} & b_{33} \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix} =$$

$$= \begin{bmatrix} b_{11}u_{1t} \\ -a_{21}b_{11}u_{1t} + b_{22}u_{2t} \\ (a_{21}a_{32} - a_{31})b_{11}u_{1t} - a_{32}b_{22}u_{2t} + b_{33}u_{3t} \end{bmatrix}$$

We can see that $\varepsilon_{1t} = b_{11}u_{1t}$, which we can substitute into the remaining two expressions for ε_{2t} and ε_{3t}. This results in an expression for ε_{2t} (= $-a_{21}\varepsilon_{1t} + b_{22}u_{2t}$), which we can solve for $b_{22}u_{2t}$ and substitute into the equation for ε_{3t}. After a little algebra (honestly!), we obtain the following three equations in the three unknowns ε_{1t}, ε_{2t} and ε_{3t}:

$$\varepsilon_{1t} = b_{11}u_{1t} \tag{18}$$

$$\varepsilon_{2t} = -a_{21}\varepsilon_{1t} + b_{22}u_{2t} \tag{19}$$

$$\varepsilon_{3t} = -a_{31}\varepsilon_{1t} - a_{32}\varepsilon_{2t} + b_{33}u_{3t} \tag{20}$$

which you can compare with the three EViews restrictions above. The correspondence between the estimated residuals, ε_t (denoted by `@e` in EViews), and the structural shocks, u_t (denoted by `@u` in EViews) should be obvious, as should the correspondence between `c(1)` (= b_{11}), `c(2)` (= $-a_{21}$), `c(3)` (= b_{22}), `c(4)` (= $-a_{31}$), `c(5)` (= $-a_{32}$) and `c(6)` (= b_{33}).

The output after imposing the restrictions on the VAR with two lags and a constant using the **text form** is given in Table 8.

Table 8: Just-identified structural VAR(2) estimates (text form)

Structural VAR Estimates
Sample (adjusted): 1981Q3 2000Q1
Included observations: 75 after adjustments
Estimation method: method of scoring (analytic derivatives)
Convergence achieved after 14 iterations
Structural VAR is just-identified

Model: Ae = Bu where E[uu']=I
Restriction Type: short-run text form

@e1 = C(1)*@u1
@e2 = C(2)*@e1 + C(3)*@u2
@e3 = C(4)*@e1 + C(5)*@e2 + C(6)*@u3
where
@e1 represents GAP residuals
@e2 represents INFL residuals
@e3 represents FF residuals

	Coefficient	Std. Error	z-Statistic	Prob.
C(2)	-0.273532	0.142629	-1.917791	0.0551
C(4)	0.485190	0.133749	3.627609	0.0003
C(5)	0.061009	0.105720	0.577080	0.5639
C(1)	0.588890	0.048083	12.24745	0.0000
C(3)	0.727397	0.059392	12.24745	0.0000
C(6)	0.665978	0.054377	12.24745	0.0000
Log likelihood	-225.1890			
Estimated A matrix:				
1.000000	0.000000	0.000000		
0.273532	1.000000	0.000000		
-0.485190	-0.061009	1.000000		
Estimated B matrix:				
0.588890	0.000000	0.000000		
0.000000	0.727397	0.000000		
0.000000	0.000000	0.665978		

What does this recursive structure mean for the small macro model due to Cho and Moreno (2006)? Ignoring the constant and employing the notation of equation (14), the original VAR(2) is given by:

$$y_t = h_{11,1}y_{t-1} + h_{12,1}\pi_{t-1} + h_{13,1}i_{t-1} + h_{11,2}y_{t-2} + h_{12,2}\pi_{t-2} + h_{13,2}i_{t-2} + \varepsilon_{1t} \quad (21)$$

$$\pi_t = h_{21,1}y_{t-1} + h_{22,1}\pi_{t-1} + h_{23,1}i_{t-1} + h_{21,2}y_{t-2} + h_{22,2}\pi_{t-2} + h_{23,2}i_{t-2} + \varepsilon_{2t} \quad (22)$$

$$i_t = h_{31,1}y_{t-1} + h_{32,1}\pi_{t-1} + h_{33,1}i_{t-1} + h_{31,2}y_{t-2} + h_{32,2}\pi_{t-2} + h_{33,2}i_{t-2} + \varepsilon_{3t} \quad (23)$$

or, in matrix notation:

$$\begin{pmatrix} y_t \\ \pi_t \\ i_t \end{pmatrix} = \begin{pmatrix} h_{11,1} & h_{12,1} & h_{13,1} \\ h_{21,1} & h_{22,1} & h_{23,1} \\ h_{31,1} & h_{32,1} & h_{33,1} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{pmatrix} + \begin{pmatrix} h_{11,2} & h_{12,2} & h_{13,2} \\ h_{21,2} & h_{22,2} & h_{23,2} \\ h_{31,2} & h_{32,2} & h_{33,2} \end{pmatrix} \begin{pmatrix} y_{t-2} \\ \pi_{t-2} \\ i_{t-2} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix} \quad (24)$$

But we are interested not in the effects of the reduced-form or observed shocks, ε_t, but in the effect of the structural shocks, u_t. But equations (18) to (20) tell us what the relationship between the ε_t's and the u_t's is. We can therefore substitute those relationships into the VAR(2) model given by equation (24):

$$\begin{pmatrix} y_t \\ \pi_t \\ i_t \end{pmatrix} = \begin{pmatrix} h_{11,1} & h_{12,1} & h_{13,1} \\ h_{21,1} & h_{22,1} & h_{23,1} \\ h_{31,1} & h_{32,1} & h_{33,1} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{pmatrix} + \begin{pmatrix} h_{11,2} & h_{12,2} & h_{13,2} \\ h_{21,2} & h_{22,2} & h_{23,2} \\ h_{31,2} & h_{32,2} & h_{33,2} \end{pmatrix} \begin{pmatrix} y_{t-2} \\ \pi_{t-2} \\ i_{t-2} \end{pmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ a_{21} & 1 & 0 \\ a_{31} & a_{32} & 1 \end{bmatrix}^{-1} \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix} \tag{25}$$

such that:

$$\begin{aligned} \begin{bmatrix} 1 & 0 & 0 \\ a_{21} & 1 & 0 \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{pmatrix} y_t \\ \pi_t \\ i_t \end{pmatrix} &= \begin{bmatrix} 1 & 0 & 0 \\ a_{21} & 1 & 0 \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{pmatrix} h_{11,1} & h_{12,1} & h_{13,1} \\ h_{21,1} & h_{22,1} & h_{23,1} \\ h_{31,1} & h_{32,1} & h_{33,1} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{pmatrix} \\ &+ \begin{bmatrix} 1 & 0 & 0 \\ a_{21} & 1 & 0 \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{pmatrix} h_{11,2} & h_{12,2} & h_{13,2} \\ h_{21,2} & h_{22,2} & h_{23,2} \\ h_{31,2} & h_{32,2} & h_{33,2} \end{pmatrix} \begin{pmatrix} y_{t-2} \\ \pi_{t-2} \\ i_{t-2} \end{pmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix} \end{aligned} \tag{26}$$

$$\begin{aligned} \begin{bmatrix} 1 & 0 & 0 \\ a_{21} & 1 & 0 \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{pmatrix} y_t \\ \pi_t \\ i_t \end{pmatrix} &= \begin{pmatrix} h_{11,1} & h_{12,1} & h_{13,1} \\ a_{21}h_{11,1} + h_{21,1} & a_{21}h_{12,1} + h_{22,1} & a_{21}h_{13,1} + h_{23,1} \\ a_{31}h_{11,1} + a_{32}h_{21,1} + h_{31,1} & a_{31}h_{12,1} + a_{32}h_{22,1} + h_{32,1} & a_{31}h_{13,1} + a_{32}h_{23,1} + h_{33,1} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{pmatrix} \\ &+ \begin{pmatrix} h_{11,2} & h_{12,2} & h_{13,2} \\ a_{21}h_{11,2} + h_{21,2} & a_{21}h_{12,2} + h_{22,2} & a_{21}h_{13,2} + h_{23,2} \\ a_{31}h_{11,2} + a_{32}h_{21,2} + h_{31,2} & a_{31}h_{12,2} + a_{32}h_{22,2} + h_{32,2} & a_{31}h_{13,2} + a_{32}h_{23,2} + h_{33,2} \end{pmatrix} \begin{pmatrix} y_{t-2} \\ \pi_{t-2} \\ i_{t-2} \end{pmatrix} \\ &+ \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix} \end{aligned} \tag{27}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ a_{21} & 1 & 0 \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{pmatrix} y_t \\ \pi_t \\ i_t \end{pmatrix} = \begin{pmatrix} \beta_{11,1} & \beta_{12,1} & \beta_{13,1} \\ \beta_{21,1} & \beta_{22,1} & \beta_{23,1} \\ \beta_{31,1} & \beta_{32,1} & \beta_{33,1} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{pmatrix} + \begin{pmatrix} \beta_{11,2} & \beta_{12,2} & \beta_{13,2} \\ \beta_{21,2} & \beta_{22,2} & \beta_{23,2} \\ \beta_{31,2} & \beta_{32,2} & \beta_{33,2} \end{pmatrix} \begin{pmatrix} y_{t-2} \\ \pi_{t-2} \\ i_{t-2} \end{pmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix} \tag{28}$$

It may not have been clear at the outset why we did all this – frankly quite tedious – matrix algebra. But let us take a closer look at the resulting model. Under the assumption of a recursive SVAR(2), we obtain:

$$y_t = \text{lagged terms in } y_t, \pi_t \text{ and } i_t + b_{11}u_{1t} \tag{29}$$

$$\pi_t = -a_{21}y_t + \text{lagged terms in } y_t, \pi_t \text{ and } i_t + b_{22}u_{2t} \tag{30}$$

$$i_t = -a_{31}y_t - a_{32}\pi_t + \text{lagged terms in } y_t, \pi_t \text{ and } i_t + b_{33}u_{3t} \tag{31}$$

Observe how these three equations now tell a story about **initial** or **short-run** responses. The output gap in equation (29) only responds to lags of itself and to lags of the other two variables. In other words, the output gap does not respond to any variable **contemporaneously**. This is in contrast to inflation, which reacts to the output gap contemporaneously (equation (30)). The size of this contemporaneous response depends on the size and magnitude of the estimated coefficient a_{21}. So the variable ordered first does not react to variables ordered below it contemporaneously, but will affect all the ones ordered below it contemporaneously. The variable ordered second responds contemporaneously to the variable above it in the ordering. Finally, we find that the variable ordered last responds contemporaneously to all the variables ordered above it or, alternatively, it does not affect the variables above it in the current period. In particular, assuming u_{1t} to be a demand shock, u_{2t} to be a supply or cost-push shock and u_{3t} to be monetary or interest-rate shock, then:

- interest rates have no effect on the output gap for one period;
- there is no **direct** effect of current interest rates upon inflation; and
- according to the interest rate rule (the last equation), the monetary authority responds only slowly to the current output gap and inflation

Structural VAR proponents try to avoid overidentifying the VAR structure and propose just enough restrictions to identify the parameters uniquely, which is what we have just done. Note that the EViews output in Table 8 explicitly mentions fact that the structural VAR is just identified. Accordingly, most SVAR models are just identified.

It is always a good idea to consider a recursive solution first, which can serve as a benchmark for later analysis. We should then ask if there is anything unreasonable about the recursive solution, which can be done by looking at the impulse response functions, say. At this stage, we can think about how the system should be modified.

An alternative approach to inputting identifying restrictions would be to use the **matrix form** restrictions. Under this approach, you would create two matrices with the following entries:

$$\mathrm{A} = \begin{pmatrix} 1 & 0 & 0 \\ NA & 1 & 0 \\ NA & NA & 1 \end{pmatrix}, \mathrm{B} = \begin{pmatrix} NA & 0 & 0 \\ 0 & NA & 0 \\ 0 & 0 & NA \end{pmatrix} \qquad (32)$$

Matrices are created by going to **Object**, **New Object…** in the workfile window and selecting **Matrix-Vector-Coef** from the list of possibilities. Make sure to give it an appropriate name – I have called them `matrix_a` and `matrix_b`. Dimension the matrix as required (in our case, we have three columns and three rows). Once the matrix comes up (all entries will be zeros), select **Edit +/-** to access the individual cells in the matrix you have created. We can then enter the individual elements of the first matrix as shown above, consisting of ones, zeros and NAs. When you are done, click on **Edit +/-** again, and then close the window. Repeat the exercise for the second matrix.

Once we have created the two matrices they will appear in the list of variables in the workfile. Return to the estimated VAR model, select the **Matrix** specification in the **Estimate Structural Factorisation…** described above, select **Short-run pattern** and enter the names of the matrices for *A* and *B* as appropriate. Except for a change in sign, the resulting output, given in Table 9, should be identical to the one obtained using the text version in Table 8 above.

Table 9: Just-identified structural VAR(2) estimates (matrix form)

Structural VAR Estimates
Sample (adjusted): 1981Q3 2000Q1
Included observations: 75 after adjustments
Estimation method: method of scoring (analytic derivatives)
Convergence achieved after 11 iterations
Structural VAR is just-identified

Model: Ae = Bu where E[uu']=I
Restriction Type: short-run pattern matrix

A =

1	0	0
C(1)	1	0
C(2)	C(3)	1

B =

C(4)	0	0
0	C(5)	0
0	0	C(6)

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.273532	0.142629	1.917791	0.0551
C(2)	-0.485190	0.133749	-3.627609	0.0003
C(3)	-0.061009	0.105720	-0.577080	0.5639
C(4)	0.588890	0.048083	12.24745	0.0000
C(5)	0.727397	0.059392	12.24745	0.0000
C(6)	0.665978	0.054377	12.24745	0.0000
Log likelihood	-225.1890			

Estimated A matrix:

1.000000	0.000000	0.000000
0.273532	1.000000	0.000000
-0.485190	-0.061009	1.000000

Estimated B matrix:

0.588890	0.000000	0.000000
0.000000	0.727397	0.000000
0.000000	0.000000	0.665978

4.2 Generating impulse response functions and forecast error variance decompositions

Two useful outputs from VARs are the impulse response function (IRF) and the forecast error variance decomposition (FEVD). An impulse response function traces the effect of a shock to one of the innovations of the VAR on current and future values of the endogenous variables. As such, a shock to the *i*-th variable directly affects the *i*-th variable contemporaneously, and is also transmitted to all of the endogenous variables with a lag through the dynamic structure of the VAR. In the case of identified SVARs, impulse responses show how the different variables in the system respond to identified, i.e., structural, shocks; in other words, they show the dynamic interactions between the endogenous variables in the VAR(p) process. Since we have 'identified' the structural VAR, the impulse responses will be depicting the responses to the **structural** shocks. In other words, once the structural model has been identified and estimated, the effects of the structural shocks, u_t, can be investigated using an impulse response analysis. We do this because the results of the impulse response analysis are often more informative than the parameter estimates of the SVAR coefficients themselves.

The same is true for forecast error variance decompositions, which are also popular tools for interpreting VAR models. While impulse response functions trace the effect of a shock to one endogenous variable onto the other variables in the VAR, forecast error variance decompositions (or variance decompositions in short) separate the variation in an endogenous variable into the contributions explained by the component shocks in the VAR. In other words, the variance decomposition tells us the proportion of the movements in a variable due to its 'own' shock versus shocks to the other variables. Thus, the variance decomposition provides information about the relative importance of each (structural) shock in affecting the variables in the VAR. In much empirical work, it is typical for a variable to explain almost all of its own forecast error variance at short horizons and smaller proportions at longer horizons. Such a delayed effect of the other endogenous variables is not unexpected, as the effects from the other variables are propagated through the reduced-form VAR with lags.

Q7. Generate impulse response functions using the Cholesky decomposition following a shock to `ff`.

Answer: Select **View** and **Impulse Response…**, which opens the **Impulse Responses** menu.[16] On the **Display** tab, select 20 periods (equal to five years), **Multiple Graphs** and **Analytic (asymptotic)** for **Response Standard Errors**. You should enter the variables for which you wish to generate innovations (**Impulses**) and the variables for which you wish to observe the responses (**Responses**). You may either enter the name of the endogenous variables or the numbers corresponding to the ordering of the variables. For example, for the three variables in our VAR (`gap`, `infl` and `ff`), you may either type:

```
gap infl ff
```

or:

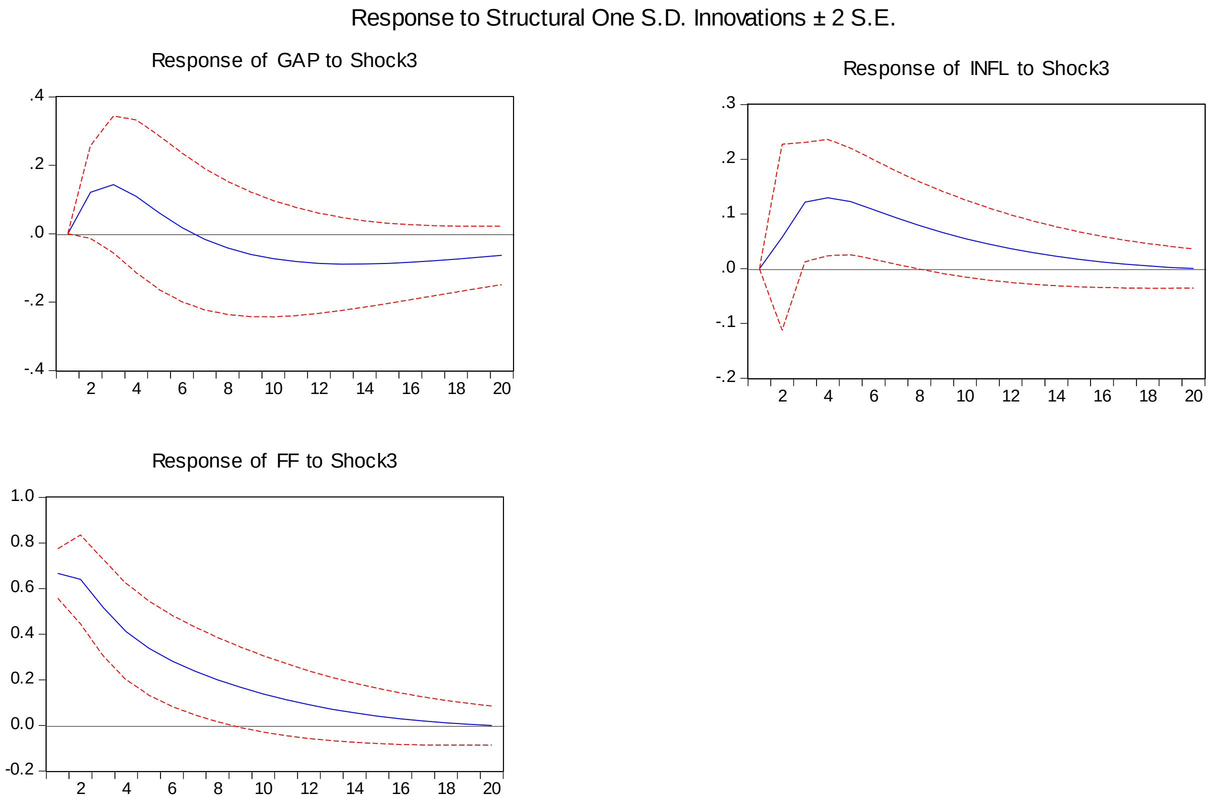
```
1 3 2
```

Note that the three numbers as entered above change the ordering of the variables, as `1 3 2` would correspond to `gap ff infl`. But the order in which you enter these variables only affects the display of the results and nothing else.

Type `ff` (or `3`) in the **Impulses** box and leave the three variables as they are in the **Responses** box. This option will show the impulse response of each variable to a structural shock to `ff` (or u_{3t}). Remember what this ordering of the three variables implies: the monetary policy shock does not affect the other two variables contemporaneously. We plot impulse responses over **20** periods. The two standard error bands of the impulse response functions are based on **analytical** (or **asymptotic**, i.e., large-sample) results. In small samples, it might be best to bootstrap the standard error bands, which can be easily done in EViews. We will do so in a moment. By clicking on the **Impulse Definition** tab, you will find that the box **Cholesky – dof adjusted** is already chosen for you – this is EViews' default option. Change this option to **Structural Decomposition**. We need to do this as we have just achieved identification by using either the text or the matrix form. Other impulse definitions can be chosen by selecting any of the other options (as we will do later on). Then click on **OK**, which should bring up Figure 3, consisting of the following three charts of impulse response functions.

[16] Alternatively, click on the **Impulse** button at the top of the VAR box.

Figure 3: Impulse response functions of `gap`, `infl` and `ff` to a one standard-deviation structural shock in `ff` (structural factorisation)



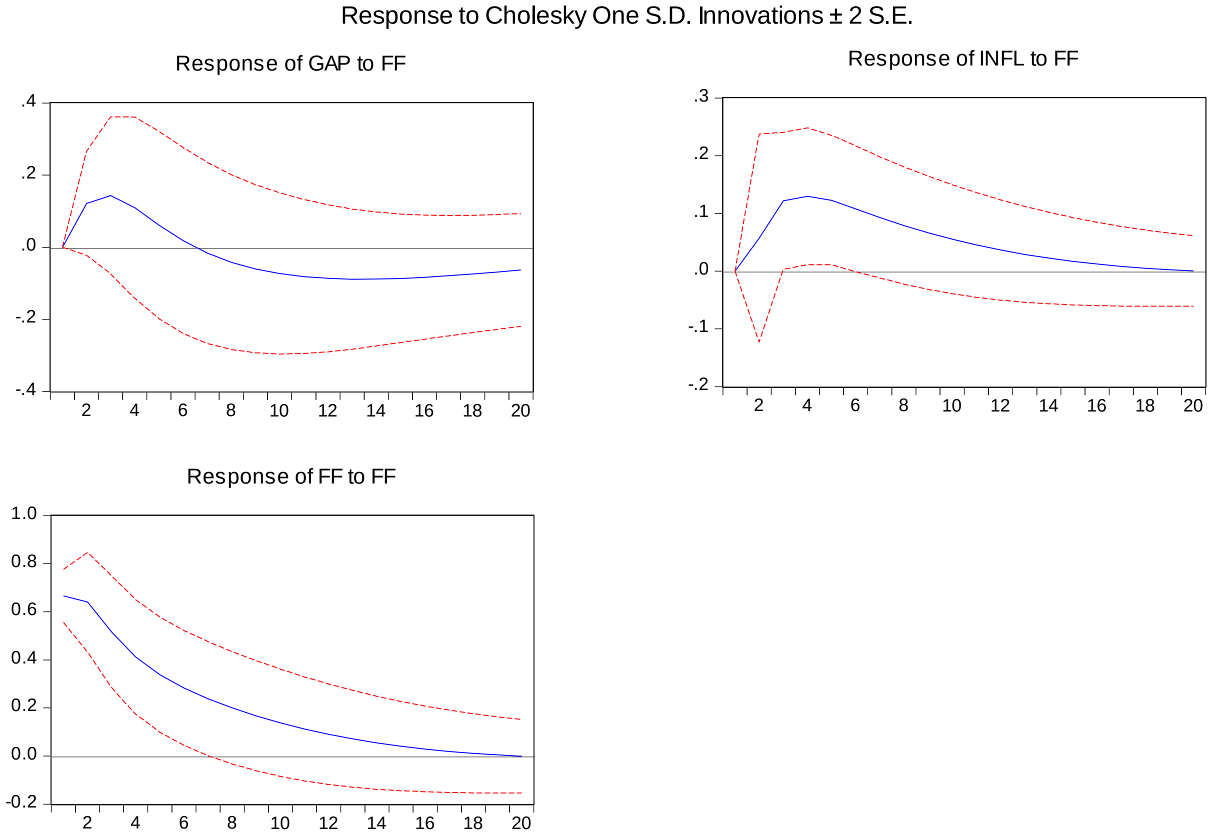
In response to a positive one-standard deviation structural shock to `ff`, the output gap first increases for some four periods before falling thereafter, inflation increases and is positive for all 20 periods (this is a manifestation of the so-called 'price puzzle' – we would not expect inflation to **increase** when we **increase**, i.e., raise the monetary policy instrument) and the policy rate increases in response to a shock to itself. Once we add the plus and minus two standard error bands, we can see how significant these effects are. The positive response of the output gap is insignificant throughout, inflation shows a significant response to `ff` between periods 3 and 8 and the positive shock of `ff` to itself persists from some nine periods.[17]

If we limit the analysis to the conventional Cholesky identification scheme, EViews allows you to plot the impulse responses of the Cholesky decomposition without having done that structural decomposition in the first place. Select **View** and **Impulse Response…**, which opens the **Impulse Responses** menu.[18] On the **Display** tab, select 20 periods and **Multiple Graphs**. In order to illustrate the bootstrapping approach to calculating standard error bands, select **Monte Carlo** with 1,000 repetitions for **Response Standard Errors**. This option shows the impulse response of each variable to shocks to the underlying fundamental shocks (u's). The two standard error bands of the impulse response functions are based on 1,000 Monte Carlo simulations. All

[17] Some of the results may be due to the choice as well as number of variables in the VAR. Taking inspiration from Sims (1992), we ought to include a forward-looking variable, such as the exchange rate, into the VAR. As a forward-looking asset price, the exchange rate can be expected to contain inflationary expectations. This should enlarge the information set of the monetary-policy maker and alleviate the price puzzle. In fact, most industrial-country benchmark VAR models of monetary policy nowadays contain six or seven variables.
[18] Alternatively, click on the **Impulse** button at the top of the VAR box.

the entries for the variables for which you wish to generate innovations (**Impulses**) and the variables for which you wish to observe the responses (**Responses**) should be appropriate, so you do not have to change anything. This option will show the impulse response of each variable to a structural shock to `ff` (or u_{3t}). By clicking on the **Impulse Definition** tab, you will find that the box **Cholesky – dof adjusted** may already be chosen for you – this is EViews' default option. If not, please select it. Results of this structural factorisation are given in Figure 4 below. As we have carried out the same structural identification scheme in two different ways, the mean impulse responses in Figure 4 should be exactly the same as the mean impulse responses in Figure 3.

Figure 4: Impulse response functions of `gap`, `infl` and `ff` to a one standard-deviation structural shock in `ff` (Cholesky decomposition)



In light of Christiano *et al.*'s (1999) three stylised facts about the effects of contractionary monetary policy shocks (the aggregate price level initially responds very little; interest rates initially rise; and aggregate output initially falls, with a *J*-shaped response and a zero long-run effect of the monetary policy shock (long-run monetary policy neutrality)), let us analyse the impact of such a shock to the short-term interest rate on the variables in the VAR. The short-term interest rate obviously increases as a result of a one-time positive shock to itself (the increase is equal to 0.67, a value we have come across before as `c(6)`, i.e., the standard deviation of the structural monetary policy shock!), but the effect of the monetary shock dies down over time. After some eight periods, the (one-time) increase in the short-term interest rate is no longer statistically significant. The output gap increases rather than falls in the short-term, but there is a zero long-run effect of the monetary policy shock. In fact, the initial output gap increase is not statistically significant either. The aggregate price level responds quite strongly – and with the opposite sign to what theory would predict. This – admittedly very basic – model of the monetary transmission mechanism shows little correspondence with Christiano *et al.*'s (1999) stylised facts of the effects of a contractionary monetary policy shock.

Q8. Generate forecast error variance decompositions for the identification scheme in our example.

Answer: Select **View** and **Variance Decomposition…** as well as the **Table** option, no standard errors and 12 periods. The following table gives the variance decomposition of the three variables in the VAR to the identified structural shocks (at two-quarter intervals to conserve space). Again, EViews allows you to calculate the forecast error variance decomposition using the Cholesky decomposition without having done that decomposition in the first place. Variance decompositions in Table 10 are given without standard errors, although you can easily use asymptotic standard errors or bootstrap them using the **Monte Carlo** option.

Table 10: Forecast error variance decompositions

Period	S.E.	GAP	INFL	FF
Variance Decomposition of GAP:				
1	0.588890	100.0000	0.000000	0.000000
2	0.985755	98.31692	0.164825	1.518251
4	1.467388	97.20042	0.605751	2.193824
6	1.693211	97.13563	1.075019	1.789353
8	1.804893	96.54759	1.816231	1.636180
10	1.863519	95.43529	2.771233	1.793477
12	1.896327	94.10308	3.770863	2.126054
Variance Decomposition of INFL:				
Period	S.E.	GAP	INFL	FF
1	0.745019	4.674657	95.32534	0.000000
2	0.802001	5.623048	93.86337	0.513580
4	0.900226	8.472654	87.21217	4.315175
6	0.956154	11.60522	81.64905	6.745727
8	0.989832	14.07236	78.11363	7.814006
10	1.010058	15.91557	75.84931	8.235120
12	1.021994	17.21977	74.41124	8.368997
Variance Decomposition of FF:				
Period	S.E.	GAP	INFL	FF
1	0.722229	14.59287	0.377556	85.02958
2	1.035208	17.97033	2.402505	79.62716
4	1.386680	22.17873	10.80182	67.01945
6	1.590871	26.23067	15.19509	58.57424
8	1.714516	29.68902	16.59435	53.71662
10	1.788103	32.36566	16.78384	50.85050
12	1.830966	34.29744	16.58603	49.11653
Cholesky Ordering: GAP INFL FF				

Of some interest is the effect of nominal on real variables, as it is sometimes found that monetary policy shocks explain only a small part of the variance of output. As such, inflation and the short-term interest rate together predict only a small percentage of the variance of the output gap, equal to some 5 per cent after 12 periods. Note how the variance decomposition of `gap` due to a shock to itself is still close to 95 per cent at the end of the observation period.

The picture is quite different for inflation, where the percentage of the variance explained by the other two variables, `ff` and `gap`, amounts to 25 per cent in period 12. Finally, after twelve periods, the short-term interest rate only explains half of its own variation, with the output gap playing an important role.

5 Imposing restrictions on individual (S)VAR coefficients

Finally, it is worth spending a bit of time investigating the individual coefficient estimates of the VAR(2) we have just estimated, which are given in Table 2. Our small-sized three-variable VAR with two lags and no deterministic regressors, which has been estimated on 75 data points, may already be overparameterised.[19] Using the empirical results on the viability of the usual asymptotic distribution of test statistics by Toda and Yamamoto (1995) and Dolado and Lütkepohl (1996), we note that just five of the 18 estimated parameters are statistically significant at the 5 per cent level (using the benchmark of the *t*-statistic being larger than 1.96 in absolute magnitude). In the VAR equation for `gap`, the statistically significant variables are `gap(-1)`, `gap(-2)` and `ff(-2)`. In the VAR equation for `infl`, the one statistically significant variable is `infl(-1)`, and in the VAR equation for `ff`, only `ff(-1)` is statistically significant.

The very real danger of using an overparameterised model, in which only a few estimated parameters are statistically significant, is that it may result in inefficient estimates and forecasts as well as poor structural analysis.[20] Specifying a parsimonious model for the data-generating process (DGP) of a set of variables is therefore a crucial step in econometric analysis. More specifically, we would like to find the 'correct' model, that is, a model which contains all the necessary right-hand side variables and is as parsimonious as possible.

5.1 Model reduction in theory

Model reduction can proceed in one of two ways, either employing model or single-equation reduction procedures.[21] Either way, the reduction strategy proceeds by imposing zero (or exclusion) restrictions on the basis of the outcomes of statistical tests. We note in passing that the existing literature on VAR model selection has traditionally mainly focused on the selection of the proper lag order, p, of an otherwise unrestricted reduced-form VAR. The choice of economic variables that enter the VAR has attracted less attention, although it may probably be the more important question. In fact, it should be obvious that there exists an interdependence between the two, as shorter lag lengths translate into fewer variables and *vice versa*.

The fact that not all parameters in an estimated VAR(p) are statistically significant raises the question of why we should have the same number of lags in each equation? Or, alternatively, can we allow VARs to have different p's in different equations, giving rise to an **unbalanced** or **subset** VAR model in either case?

Formally, a VAR model with zero constraints on the coefficients is called a **subset VAR model**, and we ought to be able to impose exclusions restrictions for at least two reasons:

- exclusion (or zero) restrictions imposed on theoretical grounds; and
- statistical evidence that some of the coefficients in the VAR(p) are zero

[19] Some rules-of-thumb on the maximum number of variables (n) and/or lag order (p) do exist. Sargan (1975), for example, suggests that $T > n(p + 1)$ in order to ensure that the likelihood function is well defined.

[20] In general, imposing restrictions on VARs is thought to aid in forecasting, as the lower the number of coefficients to estimate in the underlying VAR model, the more precise our forecast is likely to be.

[21] As we will discuss below, economic theory also frequently provide a rationale for particular exclusion restrictions.

There are a number of exclusion restrictions that arise for **theoretical** reasons. For example, it is not uncommon in NK models to hypothesise that $(1 - \delta) = 0$ in the new Keynesian Phillips curve given by equation (5) and $(1 - \mu) = 0$ in the IS equation (6). As we will see below, it is not straightforward to impose (and implement) these (cross-equation) restriction on the VAR(*p*) model given by equation (10). Furthermore, we have already discussed the fact that if there is no serial correlation in the shocks, the solution to a DSGE model is a VAR(1). It will be difficult to enforce these restrictions in this case, as `gap(-2)` is significant in the `gap` equation. You may recall our lag exclusion test in Section 4.2, where we established that the significance of `gap(-2)` in the `gap` equation makes the entire second lag significant in the VAR(2) model.

The above facts notwithstanding, it may still make sense to impose restrictions on individual coefficients within the VAR in some circumstances, such as forecasting. One way of deciding on the appropriate subset VAR model is to estimate **all** potential models and use model selection criteria to select amongst all the different possibilities. In other words, restrictions on individual parameters or groups of parameters in VARs (and VECMs) can be based on model selection criteria. The benefit of such an approach arises if there is little to no *a priori* knowledge of possible zero restrictions. This is a situation that occurs regularly. Suppose that all we know at the outset of our empirical analysis is that the maximum lag order of the VAR process is some upper limit *p*, but that no other prior knowledge of possible zero constraints is available. In this case, all we can do is to fit all possible subset VAR models, compare the different subset VAR models and choose the one which is 'optimal' under a specific (model selection or information) criterion, such as the AIC, SC or HQ.[22] In fact, subset VAR modelling frequently bases model choice on model selection criteria. Using statistical or hypothesis tests in such a situation can create problems because the different models may not be nested, meaning that statistical tests do not necessarily lead to a unique answer as to which model to use.

But it should be obvious that such a procedure can be – extremely – computationally expensive if *n*, the number of variables in the VAR, is large. Across **all** *n* equations of the VAR(*p*), there exist n^2p coefficients (not including the constant for the moment), from which:

$$\binom{n^2p}{j}$$

subsets with j ($j = 0, 1, \ldots, n^2p\text{-}1$) elements can be chosen. Thus, there is a total of:

$$\sum_{j=0}^{n^2p-1}\binom{n^2p}{j} = 2^{n^2p} - 1$$

subset VAR models, not counting the unrestricted VAR(*p*) model which is also a candidate. For instance, for our VAR(2) with three variables, there are as many as $2^{(9)(2)} - 1 = 2^{18} - 1 = 262{,}143$ subset models plus the initial unrestricted VAR(2) model.

[22] In other words, such an approach chooses the subset VAR model that optimises some prespecified information criterion. In many instances, these information criteria may have to be appropriately modified to take account of the issue of multiple comparisons. This procedure is referred to as the Bonferroni correction.

For that reason, many alternative strategies for subset VAR modelling avoid fitting all potential candidate models.[23] A different elimination procedure, in which one variable only is eliminated in each step, proceeds sequentially and is computationally more efficient. This is known as the **top-down** strategy. It starts from the full VAR(p) model and coefficients are deleted in the n equations separately. There are two different model reduction strategies, which – as shown in Brüggemann and Lütkepohl (2000) – yield the same result asymptotically, i.e., in large samples.

The first strategy proceeds on the basis of model selection criteria. The goal is to find the zero restrictions for the coefficients of the i-th equation that lead to the minimum value of a prespecified information criterion. The i-th equation of the full VAR(p) with n variables, which is given by:

$$\begin{aligned} y_{it} = \mu_i &+ \beta_{i1,1} y_{1,t-1} + \ldots + \beta_{in,1} y_{n,t-1} + \\ &\vdots \\ &+ \beta_{i1,p} y_{1,t-p} + \ldots + \beta_{in,p} y_{n,t-p} + \varepsilon_{it} \end{aligned} \tag{33}$$

is estimated by least squares and the corresponding value of the chosen information criterion is evaluated. Then the last coefficient, $\beta_{in,p}$ (i.e, the variable $y_{n,t-p}$) is deleted from the equation, and the equation is re-estimated with this restriction in place. If the value of the information criterion for the restricted model is greater than that for the unrestricted model, $y_{n,t-p}$ is retained in the equation. Otherwise, it is eliminated. Then the same procedure is repeated for the second-to-last coefficient, $\beta_{i,n-1,p}$ (or variable $y_{n-1,t-p}$), and so on. In each step, a lag of the variable is deleted if the information criterion does not increase as a result of the additional restriction compared to the smallest value obtained in the previous steps. If all coefficients can be eliminated, the testing sequence ends with μ_i as the single regressor in the i-th equation.

In a second approach, individual zero coefficients are imposed on the basis of the t-ratios of the parameter estimates. A possible strategy is to delete sequentially those regressors with the smallest absolute value of t-ratios until all t-ratios (in absolute value) are greater than some threshold value, γ. Each step of this procedure therefore eliminates a single regressor only. We then re-estimate the new – reduced – VAR model, which gives rise to a new set of t-ratios and start the evaluation process anew. One advantage of this approach is that the final outcome will not depend on the order in which the regressors are included in the model, as is the case for the top-down strategy using information criteria described above.[24] Here, the order of elimination is determined by the smallest absolute value of the t-ratio rather than the longest lag length.

5.2 Model reduction in practice

In addition to the aforementioned theoretical issues that arise in subset VAR modelling, we also have to consider some practical issues having to do with estimating subset or unbalanced VARs. Say we wanted to estimate an unbalanced or subset VAR model in which we deleted the three variables with the lowest t-ratios in absolute value. Based on the information in Table 2, this

[23] These alternative approaches for selecting subset VAR models (i.e., VARs with zero constraints on the coefficients) are usefully reviewed in Section 5.2.8 of Lütkepohl (2005), including full search, search over complete VAR matrices and top-down and bottom-up specifications of the distributed lag length.
[24] This introduces some arbitrariness into the process, as the choice of subset VAR may depend on the order in which variables are included in the initial VAR(p) model.

would mean eliminating {`ff(-1)`, `ff(-2)`} from the `infl` equation and `infl(-2)` from the `gap` equation.[25]

But it is difficult, if not to say impossible, to impose (and estimate) unbalanced VAR models within the EViews **VAR** object. EViews does not handle different lag structures in different VAR equations very well. While it is possible to take an estimated VAR object and turn it into an EViews **Model** object, this takes us out of the EViews **VAR** object, meaning that some of the extremely convenient built-in procedures, such as impulse response functions, forecast error variance decompositions and structural factorisations, will no longer be available at the push of a button. It is, however, possible to do estimation with an EViews' **System** object, which will be illustrated in Section 5.3 below. In fact, EViews itself suggests turning the VAR into a **System** object, which allows for the estimation of unbalanced systems of equations.

But by far the easiest way of handling unbalanced or subset VAR models is to estimate them with the help of an EViews program. With that purpose in mind, the following program estimates the parameters of the small recursive macroeconomic model due to Cho and Moreno (2006) and also produces impulse response functions on the basis of an unbalanced or subset VAR model. We note in passing that OLS is no longer appropriate for unbalanced VAR systems, as the individual equations will no longer have the same regressors on the right-hand side. Remember that the applicability of OLS to VAR models is due to the fact that every lagged variable appears in every equation. In an unbalanced VAR, this will no longer be the case and (full-information) **maximum likelihood** (ML) estimation will need to be applied.

A program to do this, entitled **chomoreno_lag2.prg**, can be found in the **Day 4** sub-folder of the overall **Data** folder. This EViews program is not particularly elegant, but it gets the job done in an intuitive way when applied to our model involving three variables and two lags. A program called **chomoreno_lagn.prg** is much more efficient, making use of EViews' built-in functionality. But it does so at the expense of clarity. On the plus side, it does allow for any number of variables and lags in the subset (S)VAR. Let us take a closer look at the first EViews program. I have added some comments if and when required to illustrate what is going on and what the code is intended to do.

```
' Program to estimate a variant of the recursive small macro
' model that uses Cho-Moreno (2006) data.
' This variant has no second lag of inflation in the output gap
' equation and no lags of the interest rate in the inflation
' equation.
' Because of this we will estimate the system with FIML after
' getting starting values with OLS.

smpl 1981q1 2000q1

' We are now going to do OLS on each of the three equations.
' c here is the constant term.  Later on, c will be used for a
' vector of coefficients.  This does not confuse EViews.
' In fact, it expects it.

equation iv_first.ls gap c gap(-1) gap(-2) infl(-1) ff(-1) ff(-2)
```

[25] This is done for convenience only. I am fully aware that I am contradicting my earlier statement that, after having eliminated the coefficient with the lowest (absolute) *t*-value, we should re-estimate the model with that variable being omitted from the set of regressors.

```
equation iv_second.ls infl c gap gap(-1) gap(-2) infl(-1) infl(-2)
equation iv_third.ls ff c gap infl gap(-1) gap(-2) infl(-1)
infl(-2) ff(-1) ff(-2)
```

The three lines above create linear regression equation objects with the names `iv_first`, `iv_second` and `iv_third` respectively. The dependent variable in `iv_first` is `gap`, and the independent variables are a constant, `gap(-1)`, `gap(-2)`, `infl(-1)`, `ff(-1)` and `ff(-2)`, i.e., we have already imposed the exclusion restriction that `infl(-2)` does not enter into the `gap` equation. The dependent variable in `iv_second` is `infl`, and the independent variables are a constant, `gap(-1)`, `gap(-2)`, `infl(-1)` and `infl(-2)`. We note the absence of both `ff(-1)` and `ff(-2)` from the `infl` equation, such that the exclusion restrictions are again imposed. The dependent variable in `iv_third` is `ff`, and the independent variables are a constant, `gap`, `infl`, `gap(-1)`, `gap(-2)`, `infl(-1)`, `infl(-2)`, `ff(-1)` and `ff(-2)`. The overall functional form is equivalent to a restricted version of equations (29) and (30), with equation (31) remaining unrestricted. Note the inclusion of the contemporaneous output gap variable (`gap`) in the equation for inflation as well as the contemporaneous output gap (`gap`) and inflation (`infl`) in the equation for the short-term interest rate. The contemporaneous variables enter these equations as we are modelling the individual equations of the recursive SVAR system.

Having created these three equations, we then use OLS to generate starting values for the subsequent maximum-likelihood estimation.

After running the program, we will find three equation objects in the workfile window. By double-clicking on any of them, we can open the estimated equation to inspect the estimation results.

```
' Set starting values in the c matrix.  Note that one has to
' write matplace(c,p2,7,1) where the 7 indicates that the second
' equation coefficients start in 7th place

matrix(6,1) p1
p1 = iv_first.@coefs
matplace(c,p1,1,1)
```

These lines create a (6 × 1) matrix called `p1`, whose entries are the estimated coefficients from the `iv_first` equation. Why a (6 × 1) matrix, i.e., a vector with six entries? Well, `iv_first` regresses `gap` on six independent variables, meaning that we will have six estimated coefficients. Using the `matplace(c,p1,1,1)` command, EViews then takes the `p1` vector and places it into the `c` vector, starting at row 1 and column 1. The matrices `p2` and `p3` below are created and filled along similar lines. The `matplace(c,p2,7,1)` command places the second set of (six) equation coefficients in the same `c` vector, but starting in row 7 (and column 1). In the end, all of the OLS coefficient estimates of the three equations estimated above will be contained in the `c` vector, which is of dimension (21×1) (six coefficients each from `iv_first` and `iv_second` and nine coefficients from `iv_third`).

```
matrix(6,1) p2
p2 = iv_second.@coefs
matplace(c,p2,7,1)
```

```
matrix(9,1) p3
p3 = iv_third.@coefs
matplace(c,p3,13,1)

' Now do full-information maximum likelihood (FIML) estimation

delete l_Fiml_8*
```

We delete any already existing `l_Fiml_8` log-likelihood object in the workfile object so as not to confuse EViews.

```
' Set up an EViews log-likelihood object

logl l_Fiml_8
```

We now create a **log-likelihood** object in EViews called `l_Fiml_8`, which is what the preceding line does. Then, using the `.append` syntax, we slowly build up the required log-likelihood estimation object. The next nine lines set up the appropriate inputs for maximum-likelihood estimation of `l_Fiml_8`. In particular, what goes into the `l_Fiml_8` object are three residual series, called `res1`, `res2` and `res3`; the variances of the residual series, called `d1`, `d2` and `d3`; and the functional form of the log-likelihood function. The first line in the log-likelihood specification, involving `@logl logl1`, tells EViews that the series `logl1` should be used to store the likelihood contributions. The remaining lines describe the computation of the intermediate results, such as the residuals and the standard deviations and the actual likelihood contributions.

```
l_Fiml_8.append @logl logl1

l_Fiml_8.append res1 = gap - c(1) - c(2)*gap(-1) - c(3)*gap(-2) -
c(4)*infl(-1) - c(5)*ff(-1) - c(6)*ff(-2)

l_Fiml_8.append res2 = infl - c(7) - c(8)*gap - c(9)*gap(-1) -
c(10)*gap(-2) - c(11)*infl(-1) – c(12)*infl(-2)

l_Fiml_8.append res3 = ff - c(13) - c(14)*gap - c(15)*infl -
c(16)*gap(-1) – c(17)*gap(-2) - c(18)*infl(-1) – c(19)*infl(-2) -
c(20)*ff(-1) – c(21)*ff(-1)

l_Fiml_8.append d1 = @sumsq(res1)/@obssmpl
l_Fiml_8.append d2 = @sumsq(res2)/@obssmpl
l_Fiml_8.append d3 = @sumsq(res3)/@obssmpl

' Constant omitted from likelihood

l_Fiml_8.append logl1 = -.5*log(d1*d2*d3) - .5*(((res1^2)/d1) +
((res2^2)/d2) + ((res3^2)/d3))
```

```
' Estimation must start from 1981 Q3 as two lags are dropped.
' Eviews doesn't correct for this when defining the log
' likelihood in FIML

smpl 1981q3 2000q1
```

The next two lines estimate the model defined by `l_Fiml_8` by maximum likelihood and tell EViews to display the results on screen.

```
l_Fiml_8.ml(b,m=1000,c=1.0e-08,showopts)
show l_Fiml_8
```

Now that we have obtained the estimated coefficients of the recursive small macro model, we can use this information to calculate impulse responses by hand. Remember that we cannot use EView's built-in functionality for this purpose. The individual steps below are reproduced in somewhat more detail in the Appendix.

```
' Computing impulse responses

' Set up structural matrices A0, A1 and A2 (as well as B)

matrix (3,3) a0
matrix (3,3) a1
matrix (3,3) a2
matrix (3,3) b

' Fill matrices with the requisite values.
' We fill by rows using the (fill=r) option.
' Could do this by columns using the (fill=c) option.
' Alternatively, we could write a0(1,1) = 1, a0(2,1) = -c(8)
' on separate lines.

a0.fill(by=r) 1,0,0,-c(8),1,0,-c(14),-c(15),1

a1.fill(by=r) c(2),c(4),c(5),c(9),c(11),0,c(16),c(18),c(20)

a2.fill(by=r) c(3),0,c(6),c(10),c(12),0,c(17),c(19),c(21)


' show a0
' show a1
' show a2
```

If you wanted to check whether the structural matrices were set up correctly, you simply remove the comment sign at the beginning of the preceding line, in which case EViews displays the structural matrices A_0, A_1 and A_2.

Where do these commands for A_0, A_1 and A_2 come from? Let us take another look at the three equations in the recursive model in EViews notation (these are equivalent to equations (29) to (31) above):


```
gap = c(1) + c(2)*gap(-1) + c(3)*gap(-2) + c(4)*infl(-1)
           + c(5)*ff(-1) + c(6)*ff(-2)

infl = c(7) + c(8)*gap + c(9)*gap(-1) + c(10)*gap(-2)
            + c(11)*infl(-1) + c(12)*infl(-2)

ff = c(13) + c(14)*gap + c(15)*infl + c(16)*gap(-1)
           + c(17)*gap(-2) + c(18)*infl(-1) + c(19)*infl(-2)
           + c(20)*ff(-1) + c(21)*ff(-2)
```

Collecting contemporaneous variables on the left-hand side, we get:

```
gap = c(1) + c(2)*gap(-1) + c(3)*gap(-2) + c(4)*infl(-1)
           + c(5)*ff(-1) + c(6)*ff(-2)

infl - c(8)*gap = c(7) + c(9)*gap(-1) + c(10)*gap(-2)
                       + c(11)*infl(-1) + c(12)*infl(-2)

ff - c(14)*gap - c(15)*infl = c(13) + c(16)*gap(-1)
                                    + c(17)*gap(-2)
                                    + c(18)*infl(-1)
                                    + c(19)*infl(-2)
                                    + c(20)*ff(-1)
                                    + c(21)*ff(-2)
```

Slightly abusing EViews notation, this yields the following matrix form:

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ -c(8) & 1 & 0 \\ -c(14) & -c(15) & 1 \end{pmatrix}}_{A_0} \underbrace{\begin{pmatrix} gap \\ \inf l \\ ff \end{pmatrix}}_{y_t} = \underbrace{\begin{pmatrix} c(1) \\ c(7) \\ c(13) \end{pmatrix}}_{\mu} + \underbrace{\begin{pmatrix} c(2) & c(4) & c(5) \\ c(9) & c(11) & 0 \\ c(16) & c(18) & c(20) \end{pmatrix}}_{A_1} \underbrace{\begin{pmatrix} gap(-1) \\ \inf l(-1) \\ ff(-1) \end{pmatrix}}_{y_{t-1}}$$
$$+ \underbrace{\begin{pmatrix} c(3) & 0 & c(6) \\ c(10) & c(12) & 0 \\ c(17) & c(19) & c(21) \end{pmatrix}}_{A_2} \underbrace{\begin{pmatrix} gap(-2) \\ \inf l(-2) \\ ff(-2) \end{pmatrix}}_{y_{t-2}}$$

from which we can read off the elements of A_0, A_1 and A_2 straightforwardly.

Having estimated and defined the A_0, A_1 and A_2 matrices, we proceed to calculate the impulse response functions manually. The correspondence between the estimated A_0, A_1 and A_2 matrices and the impulse response functions is briefly covered in the Appendix.

Rather than checking particular items in a nice dialog box to set the size of the structural shocks to either one standard deviation or to unity, we need to do this ourselves with the next couple of lines.

```
' If we want unit shocks put diag(b) to unity.  If standard
' deviation shocks, set diag(b) to the standard error of the
' regression from the single equations.
```

```
b(1,1) = 1
b(2,2) = 1
b(3,3) = 1
```

We now create and populate the respective short-run matrices appearing in the VAR model, which are denoted B_1 and B_2.

```
matrix(3,3) b1
matrix(3,3) b2

' Compute the VAR coefficients matrices from the
' SVAR coefficient matrices.

b1 = @inverse(a0)*a1
b2 = @inverse(a0)*a2

' Set impulse response function horizon
```

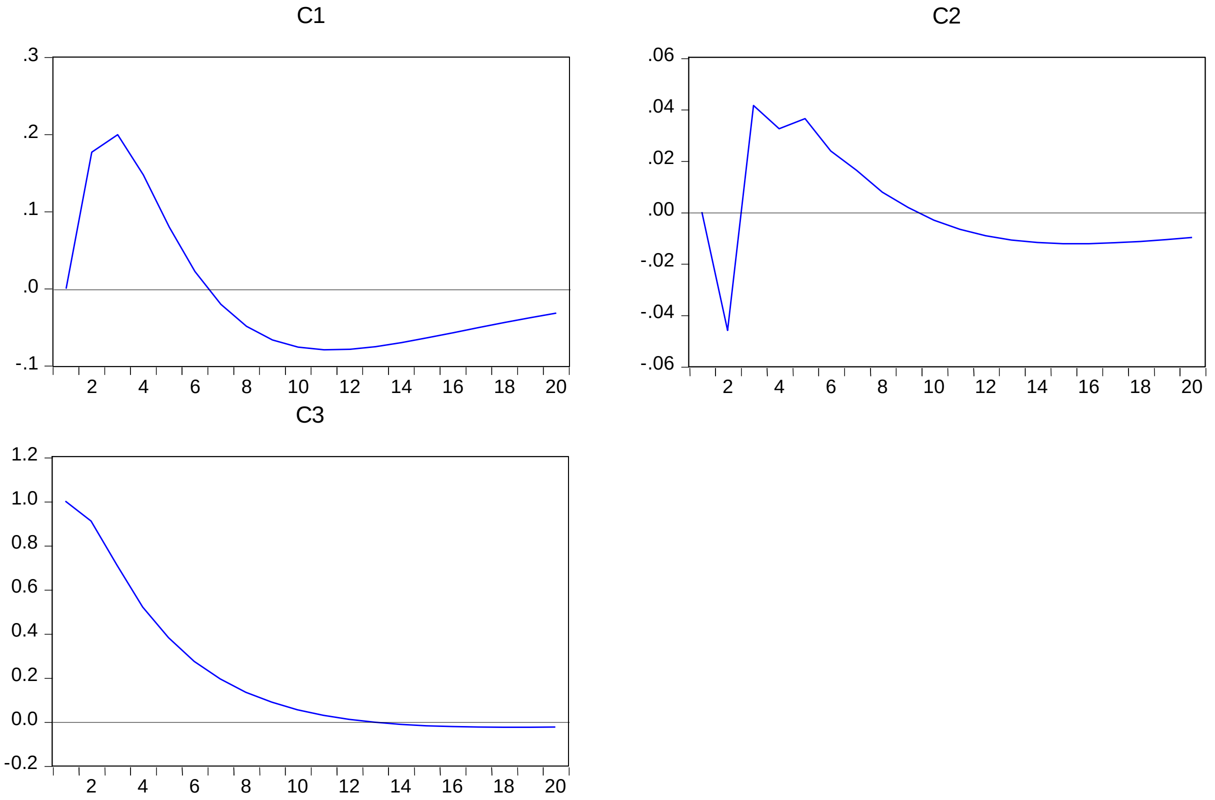
The next line allows us to set the horizon over which we want to compute impulse response functions. At the moment, we have set the horizon to 20 periods.

```
scalar horz
horz = 20

matrix (3,3) cc0

' Form initial impulse responses

cc0 = (@inverse(a0))*b

matrix(horz,3) imp1
matrix(horz,3) imp2
matrix(horz,3) imp3

' The next lines extract the impulse responses for each shock
' from the initial impulse response matrix, cc0

vector v1 = @columnextract(cc0,1)
vector v2 = @columnextract(cc0,2)
vector v3 = @columnextract(cc0,3)

for !j = 1 to 3
    imp1(1,!j) = v1(!j)
    imp2(1,!j) = v2(!j)
    imp3(1,!j) = v3(!j)
next
```

```
' Now compute one-period-ahead impulse responses.
' These follow C1 = b1*C0.

matrix(3,3) cc1
cc1 = b1*cc0

vector v1 = @columnextract(cc1,1)
vector v2 = @columnextract(cc1,2)
vector v3 = @columnextract(cc1,3)

for !j = 1 to 3
    imp1(2,!j) = v1(!j)
    imp2(2,!j) = v2(!j)
    imp3(2,!j) = v3(!j)
next

' Now compute the remainder of the impulses, which follow
' the recursion C(j) = b1*C(j-1) + b2*C(j-2) outlined in the
' Appendix

matrix(3,3) cc2

for !kk = 3 to horz

cc2 = b1*cc1 + b2*cc0

vector v1 = @columnextract(cc2,1)
vector v2 = @columnextract(cc2,2)
vector v3 = @columnextract(cc2,3)

for !j = 1 to 3
    imp1(!kk,!j) = v1(!j)
    imp2(!kk,!j) = v2(!j)
    imp3(!kk,!j) = v3(!j)
next

cc0 = cc1
cc1 = cc2

next

' Impulse responses can be found in imp1, imp2 and imp3.
' Print out responses to third shock.  The rows are periods ahead
' and the columns are variables.

show imp3
```

After running the above program, Figure 5 shows the impulse response functions of `gap`, `infl` and `ff` to a one unit shock in `ff`. They are based on the subset VAR as well as a recursive (Cholesky) decomposition of the reduced-form residuals.

Figure 5: Impulse response functions of `gap`, `infl` and `ff` to a one unit structural shock in `ff` (unbalanced SVAR, Cholesky decomposition)



In Figure 5, `C1` represents the output gap, `C2` the inflation rate and `C3` the short-term interest rate. According to the impulse-response functions in this unbalanced SVAR, output shows a one-quarter delayed increase in response to a contractionary, i.e., positive monetary policy shock, which runs counter to the first of Christiano *et al.*'s (1990) stylised facts about the effects of a contractionary monetary shock. Turning to inflation, prices first fall and then rise following an increase in interest rates. In fact, this initial dip in inflation is the main difference between the impulse responses given in Figures 3 and 4 and those given in Figure 5.

5.3 Estimating systems by (full-information) maximum likelihood in EViews*

In general, a system is a group of equations containing unknown parameters. Systems can be estimated using a number of multivariate techniques (OLS, weighted least-squares, seemingly unrelated regression, (weighted) two-stage least-squares (2SLS), three-stage least-squares (3SLS), full-information maximum likelihood and GMM) that take into account the interdependencies among the equations in the system. Methods such as OLS estimate each equation in the system separately. A second approach is to estimate the complete set of parameters of the equations in the system simultaneously. The simultaneous approach allows us to employ techniques that account for simultaneity as well as correlation in the residuals across equations and to place constraints on coefficients across equations.

Before we can apply any of the system-estimation methodologies, we have to create and specify a **system** first. To estimate the parameters of a system in EViews, we need to create a **system** object and specify the system of equations. Click on **Object/New Object…/System**, which opens the system object window. Alternatively, you can type `system` in the command window. When you first create the system, the window will be blank. In other words, you will need to fill the system specification window with text describing the equations and, depending on the estimation methodology, lines describing the instruments and – possibly – parameter starting values.[26]

Enter your equations as formulae using standard EViews expressions. Do not forget that the equations in your system should be behavioural (stochastic) equations with unknown coefficients (using EViews' default coefficient notation `c(1)`, `c(2)`, etc.) and an implicit error term.[27] The three equations of our recursive small macro model are:

```
gap = c(1) + c(2)*gap(-1) + c(3)*gap(-2) + c(4)*infl(-1)
           + c(5)*ff(-1) + c(6)*ff(-2)

infl = c(7) + c(8)*gap + c(9)*gap(-1) + c(10)*gap(-2)
            + c(11)*infl(-1) + c(12)*infl(-2)

ff = c(13) + c(14)*gap + c(15)*infl + c(16)*gap(-1)
           + c(17)*gap(-2) + c(18)*infl(-1) + c(19)*infl(-2)
           + c(20)*ff(-1) + c(21)*ff(-2)
```

As an aside, note that EViews can also automatically generate **linear** equations in a system from a list of variables. To use this automatic procedure, first highlight the dependent variables that will be in the system. Next, double-click on any of the highlighted series and select **Option/Open System…** or right-click and select **Open/as System…**. The **Make System** dialog box should appear with the variable names entered in the **Dependent variables** field. You may need to adjust the system that EViews has created for you. Moreover, you can augment the specification by adding regressors or AR(•) terms to account for serial correlation in the residuals, either estimated with common or equation-specific coefficients.

Because FIML involves an iterative process, we need to supply EViews with starting values. Just as above, we can use the OLS estimates of the three individual equations for that purpose. The complete system for the three model equations is therefore given by:

```
param c(1) 0.03409 c(2) 1.23392 c(3) -0.31012 c(4) -0.07539 c(5)
0.17707 c(6) -0.18301 c(7) -0.11322 c(8) -0.25998 c(9) 0.63745
c(10) -0.32717 c(11) 0.42319 c(12) 0.35327  c(13) -0.16618 c(14)
0.49421 c(15) 0.02584 c(16) -0.41691 c(17) 0.03082 c(18) 0.14470
c(19) 0.19188 c(20) 0.81980 c(21) -0.06051

gap = c(1) + c(2)*gap(-1) + c(3)*gap(-2) + c(4)*infl(-1)
           + c(5)*ff(-1) + c(6)*ff(-2)
```

[26] Starting values for some or all of the parameters are really only necessary for systems that contain non-linear equations.
[27] If an equation does not have an error term, it is an identity, and should not be included in the system to be estimated.

```
infl = c(7) + c(8)*gap + c(9)*gap(-1) + c(10)*gap(-2)
            + c(11)*infl(-1) + c(12)*infl(-2)

ff = c(13) + c(14)*gap + c(15)*infl + c(16)*gap(-1)
           + c(17)*gap(-2) + c(18)*infl(-1) + c(19)*infl(-2)
           + c(20)*ff(-1) + c(21)*ff(-2)
```

Make sure to give your system a name by pressing the **Name** button and choosing an appropriate name. In my case, I have named the system `cho_moreno_sys`. Then click on the **Estimate** button on the toolbar to bring up the **System Estimation** dialog. The drop-down menu marked **Estimation Method** provides you with several options for the estimation method. We will be using the **Full Information Maximum Likelihood** option.

To specify the methods used in iteration, click on the **Options** tab. To bring the FIML estimation in the system object in line with our earlier ML estimation, we change the **Optimisation algorithm** from **Marquardt** to **Berndt-Hall-Hall-Hausman** and set **Max Iterations** to 1000 and **Convergence** to 1e-08. Pressing **OK** yields the result in Table 11.

Table 11: Full information maximum likelihood estimates of the Cho and Moreno (2006) macro model

System: CHO_MORENO_SYS
Estimation Method: Full Information Maximum Likelihood (BHHH)
Sample: 1981Q3 2000Q1
Included observations: 75
Total system (balanced) observations 225
Failure to improve Likelihood after 137 iterations
WARNING: Singular covariance - coefficients are not unique

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.034165	NA	NA	NA
C(2)	1.243215	NA	NA	NA
C(3)	-0.316473	NA	NA	NA
C(4)	-0.047788	NA	NA	NA
C(5)	0.154039	NA	NA	NA
C(6)	-0.175503	NA	NA	NA
C(7)	-0.111256	NA	NA	NA
C(8)	-0.590022	NA	NA	NA
C(9)	1.067841	NA	NA	NA
C(10)	-0.448484	NA	NA	NA
C(11)	0.392402	NA	NA	NA
C(12)	0.352469	NA	NA	NA
C(13)	-0.166203	NA	NA	NA
C(14)	0.494225	NA	NA	NA
C(15)	0.025650	NA	NA	NA
C(16)	-0.416864	NA	NA	NA
C(17)	0.030778	NA	NA	NA
C(18)	0.144782	NA	NA	NA
C(19)	0.191944	NA	NA	NA
C(20)	0.819779	NA	NA	NA
C(21)	-0.060489	NA	NA	NA

Log likelihood	-214.4173	Schwarz criterion	6.926690
Avg. log likelihood	-0.952966	Hannan-Quinn criter.	6.536891
Akaike info criterion	6.277793		
Determinant residual covariance		0.061064	

Equation: GAP = C(1) + C(2)*GAP(-1) + C(3)*GAP(-2) + C(4)*INFL(-1) + C(5) *FF(-1) + C(6)*FF(-2)
Observations: 75

R-squared	0.930980	Mean dependent var	-0.046347
Adjusted R-squared	0.925979	S.D. dependent var	2.165720
S.E. of regression	0.589223	Sum squared resid	23.95568
Durbin-Watson stat	1.906521		

Equation: INFL = C(7) + C(8)*GAP + C(9)*GAP(-1) + C(10)*GAP(-2) + C(11) *INFL(-1) + C(12)*INFL(-2)
Observations: 75

R-squared	0.692264	Mean dependent var	-0.154104
Adjusted R-squared	0.669964	S.D. dependent var	1.331185
S.E. of regression	0.764749	Sum squared resid	40.35401
Durbin-Watson stat	2.100495		

Equation: FF = C(13) + C(14)*GAP + C(15)*INFL + C(16)*GAP(-1) + C(17) *GAP(-2) + C(18)*INFL(-1) + C(19)*INFL(-2) + C(20)*FF(-1) + C(21)*FF(-2)
Observations: 75

R-squared	0.951639	Mean dependent var	-0.265200
Adjusted R-squared	0.945777	S.D. dependent var	2.832899
S.E. of regression	0.659665	Sum squared resid	28.72042
Durbin-Watson stat	1.853622		

Leaving aside the (inconvenient) fact that we cannot obtain standard errors due to the singular covariance matrix, we find that the FIML estimates are little changed from the earlier ML estimates that we obtained with the EViews program. This consistency across the two different sets of results is actually very good news. The estimation results in Section 5.2 implicitly assume that the residuals of the model constitute structural shocks. These shocks are not very typical, as residuals across equations are typically contemporaneously correlated. Typically, the variance-covariance matrix of structural shocks is specified as a **diagonal** matrix, because the shocks are assumed to originate from independent sources. They are hence purely **exogenous** and **mutually uncorrelated**.

In contrast, the full information maximum likelihood estimates in Table 11 allows for contemporaneous correlation of residuals across equations. The fact that the two sets of estimates are fairly similar indicates that the assumption of mutually uncorrelated residuals probably holds in the data set under investigation, i.e., the three residual series really do constitute structural, rather than reduced-form, shocks.

6 The small open economy assumption and block exogeneity

Most empirical modelling proceeds under the assumption that the economy under investigation is a small open economy, meaning that rest-of-the-world variables are treated as exogenous. This throws up some interesting challenges in VAR modelling, where **all** variables are treated as potentially endogenous. At the same time, we would still like to include some external variables, such as the exchange rate and foreign monetary policy variables, into the VAR model, as the domestic monetary authority may keep a watchful eye on developments in these variables. Since these variables are frequently known to the domestic policymakers when they make decisions, it is natural that the monetary authority would contemporaneously respond to those home and foreign variables. At the same time, being a small open economy, decisions of the domestic monetary authority as well as the private sector are themselves unlikely to have any effects on the rest of the world. In practical terms, therefore, it might be more plausible to let the foreign block of variables follow a process that is exogenous from the small open economy's point of view.

In addition, topics such as identifying exogenous monetary policy shocks in an open economy can lead to substantial complications relative to the closed-economy case, and papers that have examined the effects of monetary policy shocks in open economies have been distinctly less successful in providing accepted empirical evidence than the VAR approach in closed economies.

For example, Cushman and Zha (1997) were able to demonstrate that previous puzzling results in the open-economy context were due to the mechanical application of the recursive identification scheme. They found that a more careful identification of monetary policy in an explicit open economy setting made the often puzzling exchange-rate response more consistent with existing open economy macroeconomic theory.

The gist of their approach is that, rather than relying solely on the recursive Cholesky technique, it allows for simultaneous interactions between the domestic monetary policy variable and other macroeconomic variables within the same period. Their approach was to impose two blocks of structural equations, where one block represents the structural equations of the international economy and the other block represents the structural equations of the domestic economy. The main implication of the small open economy assumption is that dependent variables in the domestic economy block are completely absent from the equations in the international block.

There are several advantages to be gained from this block exogeneity approach for the small open economy. One advantage highlighted by Cushman and Zha (19997), as well as Kim and Roubini (2000), is that it helps identify a monetary policy reaction function for the small open economy, as the approach enables monetary policy to react contemporaneously to a variety of domestic and international variables whose data are likely to be immediately available to the monetary authority.[28]

Moreover, another advantage of the block exogeneity assumption is that it allows for a larger set of international variables to be included in the model, while reducing the number of parameters needed to estimate the domestic block.[29]

Following the notation in Zha (1996), we assume that the structural model is given by an SVAR, such that:

$$A(L)y_t = u_t \tag{34}$$

where $A(L)$ is a $(n\times n)$ matrix polynomial in the lag operator L, y_t is a $(n\times 1)$ vector of observations on n variables and u_t is a $(n\times 1)$ vector of iid structural shocks with $E[u_t] = 0$ and $E[u_t u_t'] = I_n$.[30]

To see how the SVAR given by equation (34) can be used to model a small open economy, break equation (34) into two blocks, one representing the (small) domestic economy, y_{1t}, and the other representing the foreign (rest of the world) economy, y_{2t}. More specifically, we move to partioned matrices and let:

[28] Kim and Roubini (2000), for example, included the contemporaneous exchange rate and the world price of oil (as a proxy for expected inflation) in the domestic monetary policy reaction function. Cushman and Zha (1997) included the contemporaneous US federal funds rate in their specification of the domestic monetary policy reaction function.

[29] Cushman and Zha (1997), for example, included four international variables in their model of the Canadian economy: US industrial production, US consumer prices, the US federal funds rate and world total commodity export prices.

[30] The reduced form of equation (34) can be obtained by multiplying through by A_0^{-1}.

$$A(L) = \begin{pmatrix} A_{11}(L) & A_{12}(L) \\ A_{21}(L) & A_{22}(L) \end{pmatrix} \tag{35}$$

and:

$$y_t = \begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} \tag{36}$$

and:

$$u_t = \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} \tag{37}$$

In equation (34), the matrix A_j is the coefficient matrix on L^j in $A(L)$, where L^j is the lag operator raised to the power j. In our analysis so far, the (short-run) identification restrictions have been imposed on A_0, i.e., the contemporaneous coefficient matrix.

The small open economy assumption in Cushman and Zha (1997) is achieved by imposing the restriction that $A_{21}(L) = 0$, meaning that the small country takes changes in foreign economic conditions as given or exogenous. In practical terms, this means that the foreign block of variables is treated as exogenous from the domestic economy's point of view not only contemporaneously, but also for the other lagged values.

The following empirical illustration should serve to explain the modelling task in more detail.

6.1 Illustration: the case of Canada

The aim of this part of the exercise is to estimate different VARs for Canada, a quintessentially small open economy and identify structural shocks (such as monetary policy shocks) by imposing appropriate short-run restrictions using EViews.

Our data are contained in the EViews workfile **us and canada.wf1**, which comprises the following data series for Canada (denoted by the suffix '`_cn`') and the US (denoted by the suffix '`_us`'):[31]

- base rates (`br_cn` and `br_us`);
- the three-month Canadian Treasury Bill interest rate (`treasuryb_cn`);
- an average three- to five-year Canadian government bond yield (`gyield_cn`);
- nominal Canadian M1, M2 and M3 (`m1_cn`, `m2_cn` and `m3_cn`);
- consumer price indices (`cpi_cn` and `cpi_us`);
- GDP deflators (`gdpdef_cn` and `gdpdef_us`);
- real GDPs (`rgdp_cn` and `rgdp_us`);
- real Canadian gross fixed capital formation (`rinv_cn`);
- nominal and real effective exchange rates (`nom_eer_cn` and `nom_eer_us` as well as `re_cn` and `re_us`);

[31] But we will not be using all of the data series in the following analysis.

- the volume of Canadian exports and imports (x_cn and im_cn);
- the Canadian unemployment rate (un_cn); and
- a commodity price index (pcm).

We constructed other series such as inflation (inf_cn and inf_us), the interest-rate differential between Canda and the US (br_cn_us), the exchange rate depreciation (dpn_cn and dpn_us), the output gap using the Hodrick-Prescott filter (gdp_gap_cn and gdp_gap_us) and the annual growth rate of GDP (dgdp4_cn and dgdp4_us). Variables that start with an 'l' denote the logarithm of the series.

The data are quarterly and span the period from 1950 Q1 to 2005 Q2, although not all series are available for such a long period. Unless we are confident in assuming that the underlying monetary policy shocks are robust to different monetary policy regimes (money-supply targeting, exchange-rate targeting, inflation targeting, etc.) and changes in regimes over time, it is important to estimate parameters in SVARs on a **single** policy regime. Any regime shift therefore requires a different parameterisation of the SVAR model. This important *caveat* may explain some of the counterintuitive results we will encounter in the following exercises, in which VARs are estimated and SVARs identified over long time periods.[32]

To open the EViews workfile from within EViews, choose **File**, **Open, EViews Workfile…**, select **us and canada.wf1** from the appropriate folder and click on **Open**. Alternatively, you can double-click on the workfile icon outside of EViews, which will open EViews automatically.

Q9. Canada is a small open economy neighbouring the much larger US. To reflect this, we add a US block. To add the US components, we include US variables of interest to our VAR as either endogenous or exogenous variables.

Answer: For the endogenous VAR using the recursive approach, estimate a VAR with the variables in the following order: lrgdp_us, lcpi_us, br_us, pcm, br_cn, treasury_cn, lnom_eer_cn, lrgdp_cn and lcpi_cn; in other words, we order the US and rest of the world variables first (why?). Results of the VAR lag order selection criteria are reproduced in Table 12.

Table 12: VAR lag order selection criteria

Endogenous variables: LRGDP_US INF_US BR_US PCM BR_CN TREASURYB_CN LNOM_EER_CN LRGDP_CN INF_CN
Exogenous variables: C
Sample: 1950Q1 2005Q2
Included observations: 168

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-1579.862	NA	0.001326	18.91503	19.08238	18.98295
1	437.7503	3795.033	1.29e-13	-4.139885	-2.466333*	-3.460676
2	580.8440	253.8209	6.20e-14	-4.879095	-1.699346	-3.588598*
3	648.5098	112.7764	7.41e-14	-4.720355	-0.034409	-2.818570
4	736.6076	137.3906	7.07e-14	-4.804852	1.387291	-2.291779
5	843.0536	154.6002	5.57e-14*	-5.107781	2.590558	-1.983420

[32] For example, many structural VAR studies of US monetary policy leave out the disinflationary period from 1979 to 1984, which constitutes a different monetary policy regime.

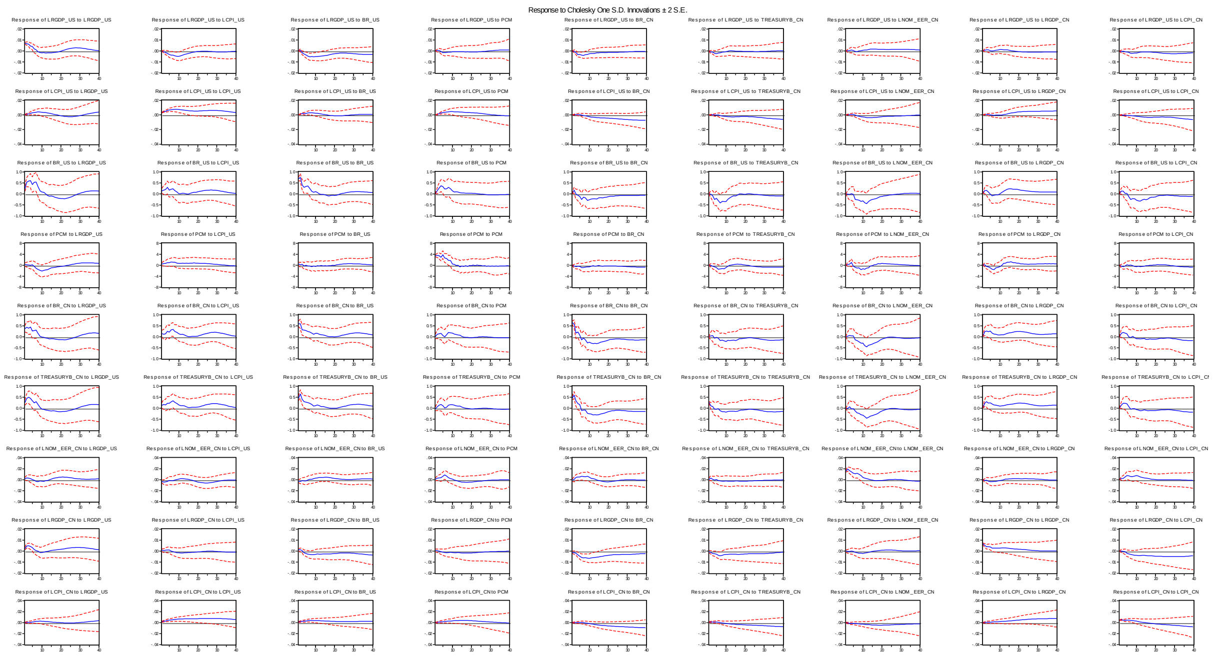
6	931.0270	118.3451	5.64e-14	-5.190798	4.013739	-1.455148
7	1014.332	103.1397	6.30e-14	-5.218240	5.492494	-0.871303
8	1107.152	104.9747*	6.64e-14	-5.358951*	6.857979	-0.400726

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Information criteria suggest lag lengths between one and eight, but you will find that we require six lags to purge autocorrelation up to sixth order from the residuals. The VAR is stable but the residual properties are non-normal.

Barely legible impulse response functions for the variables in the system are shown in Figure 6.

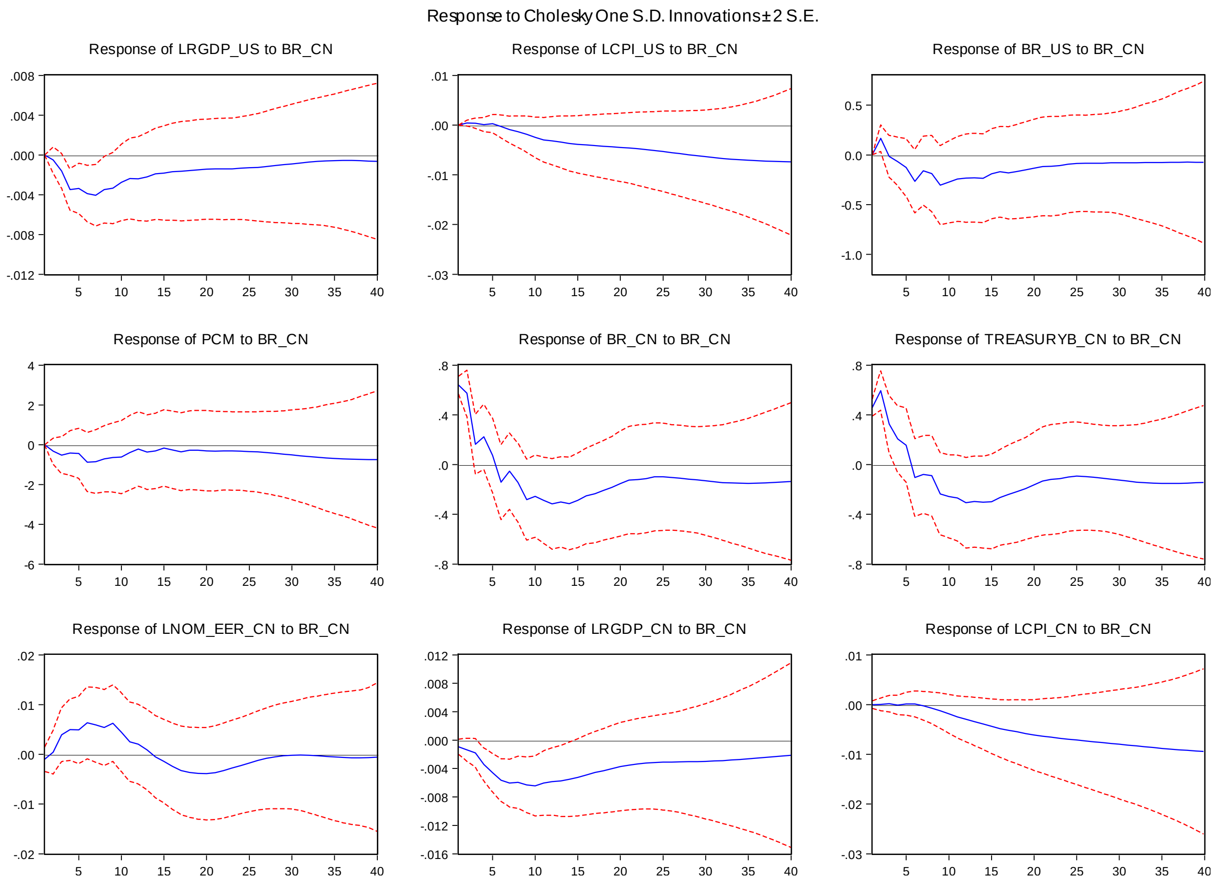
Figure 6: Impulse response functions to a one standard-deviation structural shock (Cholesky decomposition)



A number of observations are worth mentioning and there are some interesting results. In cases of non-normality, we have to be careful about the interpretation of the results, as asymptotic confidence intervals are likely to be suspect. But we have bootstrapped our confidence intervals, so we can be more confident about interpreting the results. US variables do appear to affect some Canadian variables: higher prices in the US lead to higher prices in Canada, and while higher output in the US does not increase Canadian prices, higher output in the US temporarily increases output in Canada. It is interesting to note that higher US GDP and higher US interest rates increase interest rates in Canada in the short-run, suggesting that the Canadian monetary authorities may be looking at some US variables. The exchange rate seems to react only to its own shock.

In order to make more sense of Figure 7, we can focus on the impulse response functions of the variables to just one shock, namely the Canadian monetary policy instrument (`br_cn`).

Figure 7: Impulse response functions
to a one standard-deviation structural shock in `br_cn` (Cholesky decomposition)



How do the results change if we now allow for block exogeneity? What was a single vector of Canadian and US macroeconomic variables consisting of y = (`br_cn`, `treasuryb_cn`, `lnom_eer_cn`, `lrgdp_cn`, `lcpi_cn`, `lrgdp_us`, `lcpi_us`, `br_us`, `cpm`) will now be broken up into two separate blocks of variables: the Canadian block of variables is given by y_1 = (`br_cn`, `treasuryb_cn`, `lnom_eer_cn`, `lrgdp_cn`, `lcpi_cn`), while the US block of variables is given by y_2 = (`lrgdp_us`, `lcpi_us`, `br_us`, `pcm`). Note that we have reversed the order of the countries when moving to the alternative identification scheme. In **Question 9**, we put the US variables before the Canadian variables, while we now switch the order of variables.

For ease of explanation, I re-write the SVAR given by equation (34) as:

$$Ay_t = Fz_t + u_t \tag{38}$$

where $z_t = (y_{t\text{-}1}, \ldots , y_{t\text{-}p})'$ and $F = (B_1 \ldots B_p)$, with z_t being a (np×1) column vector of all lagged variables and F being the (n×np) matrix of all lagged coefficients. Including the block exogeneity assumption into the SVAR given by equation (38) yields for the partitioned matrices:

$$\begin{pmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{pmatrix}\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} F_{11} & F_{12} \\ 0 & F_{22} \end{pmatrix}\begin{pmatrix} z_{1t} \\ z_{2t} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} \tag{39}$$

Looking at equation (39) makes it obvious that we need to impose restrictions on both the contemporaneous impact matrix as well as the lagged coefficient matrices. More specifically, the restriction that $A_{21} = 0$ follows from the assumption that the Canadian block of variables does not enter into the non-Canadian block contemporaneously, while the restriction that $F_{21} = 0$ follows from the assumption that the Canadian block of variables does not enter into the non-Canadian block with lags. We note in passing that the concept of block exogeneity is similar to Sims' (1980) concept of Granger-causality in the reduced-form VAR model.

The reduced-form of the SVAR model given by equation (39) can be written as:

$$\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix} \begin{pmatrix} z_{1t} \\ z_{2t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \tag{40}$$

or:

$$y_t = Ez_t + \varepsilon_t \tag{41}$$

where $E = A^{-1}F$ and $\varepsilon_t = A^{-1}u_t$. The block exogeneity assumptions that arise from equation (39) can be imposed on the reduced-form model (41), such that they imply $E_{21} = 0$. This is convenient as the restriction is equivalent to the testable hypothesis that z_{2t} is Granger causally prior to y_{1t} as pointed out by Sims (1980). In essence, the restrictions $E_{21} = 0$ imply that all of the estimated coefficients on lags of Canadian variables in the equations for non-Canadian variables are zero.

In the current example, we would have to check whether the coefficients on the six lags of all five Canadian variables in the equations for the found non-Canadian variables are all jointly zero. This is most conveniently set up with the help of a small EViews program.

–– To follow ––

The contemporaneous impact matrix, A_0, embodies a number of assumptions. To begin with, we assume that the Bank of Canada reacts contemporaneously some foreign variables in addition to some key domestic macroeconomic variables. With the exception of the foreign variables, which are assumed to follow an exogenous process in order to maintain the small open-economy assumption, these variables also respond to the monetary policy variable within the month. The identification scheme also defines a monetary policy reaction function involving the contemporaneous values of the exchange rate, the foreign monetary policy rate and the lagged values of all variables in the model.[33] Assuming that financial variables react instantaneously with each other, I also allow the nominal interest rate to react contemporaneously to the financial variables in the model. In addition, we assume that the foreign-exchange market is operating efficiently and with no information lags, such that all relevant information is available within the period. This translates into the exchange rate reacting to all the variables in the model contemporaneously. Since shocks from Canada have little to no effect on the rest of the world, the foreign block of variables is treated as exogenous from Canada's point of view, which explains the (4×5) matrix of zeroes in the bottom left-hand of the matrix A_0 in equation (42) below.

[33] Taking inspiration from Sims (1992), we ought to include a forward-looking variable, such as the exchange rate or a commodity price series, into the VAR. The reason for this is that a forward-looking asset price can be expected to contain a measure of inflationary expectations. This should enlarge the information set of the monetary-policy maker and alleviate the price puzzle.

Overall, this will change the structure of the short-run contemporaneous impact matrix, A0, from a recursive to a non-recursive structure. Recall from the presentation that it is only the overall number of identifying restrictions that matters; there is therefore nothing that necessarily requires identification restrictions to follow the Wold causal chain, i.e., a recursive structure. We should note that all the assumptions embodied in A_0 can be criticised on theoretical grounds.[34]

The relationship between the reduced-form residual, ε_t, and the structural shocks, u_t, is shown in equation (42). We should recall from the presentation that these contemporaneous restrictions do not merely describe the relationships between the estimated residuals and the structural shocks but also the contemporaneous relationships among the levels of the variables (remember equations (29) to (31) above?). For that reason, each line in equation (42) shows the contemporaneous relationships of a variable with the other variables in the model.

For example, the first line of equation (42) contains the monetary policy reaction function, in which the monetary policy instrument (`br_cn`) is a function of the nominal interest rate (`treasury_cn`), the nominal exchange rate (`lnom_eer_cn`), the federal funds rate (`br_us`) and a commodity price index (`pcm`).

As discussed above, an important feature of the structural identification is the simultaneous interaction of financial variables with each other. As such, the second line in equation (42) represents a nominal interest rate equation according to which the (short-term) nominal interest rate responds contemporaneously to the monetary policy instrument (`br_cn`), the exchange rate (`lnom_eer_cn`) and the federal funds rate (`br_us`). The last three lines of equation (42) underscore how the foreign variables (`lrgdp_us`, `lcpi_us` and `br_us`) are only contemporaneously affected by shocks to itself and not by shocks emanating from the domestic economy.

Finally, the production sector of the Canadian economy is determined by two variables, real output (`lrgdp_cn`) and inflation (`lcpi_cn`). Lines 4 and 5 of equation (42) reveal that domestic and foreign financial variables do not affect real activities contemporaneously, but only with a lag.

$$\begin{pmatrix} u_{i0} \\ u_i \\ u_s \\ u_y \\ u_\pi \\ u_{y*} \\ u_{\pi*} \\ u_{i*} \\ u_{com} \end{pmatrix} = \begin{pmatrix} 1 & a_{12} & a_{13} & 0 & 0 & 0 & 0 & a_{18} & a_{19} \\ a_{21} & 1 & a_{23} & 0 & 0 & 0 & 0 & a_{28} & 0 \\ a_{31} & a_{32} & 1 & a_{34} & a_{35} & a_{36} & a_{37} & a_{38} & a_{39} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{54} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{76} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{86} & a_{87} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{96} & a_{97} & a_{98} & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{i0} \\ \varepsilon_i \\ \varepsilon_s \\ \varepsilon_y \\ \varepsilon_\pi \\ \varepsilon_{y*} \\ \varepsilon_{\pi*} \\ \varepsilon_{i*} \\ \varepsilon_{com} \end{pmatrix} \qquad (42)$$

[34] We note in passing the results in Christiano *et al.* (2006), who demonstrated that if the relevant short-run identifying restrictions are justified, the structural VAR procedure reliably recovers and identifies the dynamic effects of shocks to the economy. In a similar vein, Fernandez-Villaverde *et al.* (2007) also demonstrated that if the variables chosen by the econometrician are measured without error and if the identifying restrictions are valid, the impulse response functions of the VAR model accurately represent the dynamic behaviour of the macroeconomic variables due to the shock.

In order to implement the Cushman and Zha (1997) methodology, we can use the blueprint of the earlier EViews program for the unbalanced or subset VAR model.

—— To follow ——

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Appendix: Moving from the VAR to the VMA

Given the lag length, p, we can estimate the elements of the B_j coefficient matrices of the VAR(p) by OLS and then invert the VAR(p) to get the **Wold vector moving-average (VMA) representation**:

$$y_t - \mu = \left(I_n - \sum_{j=1}^{p} B_j L^j \right)^{-1} \varepsilon_t = \varepsilon_t + C_1 \varepsilon_{t-1} + C_2 \varepsilon_{t-2} + \ldots = \sum_{j=0}^{\infty} C_j L^j \varepsilon_t \qquad \text{(A.1)}$$

where $C_0 = \mathrm{I}_n$ (the ($n \times n$) identity matrix) and L is the lag operator such that $L^j X_t = X_{t-j}$ for any variable X.

To solve for the C_j matrices, we equate the coefficients on powers of the lag operator. From (A.1) we know that:

$$\begin{aligned} \sum_{j=0}^{\infty} C_j L^j \varepsilon_t &= \left(I_n - \sum_{j=1}^{p} B_j L^j \right)^{-1} \varepsilon_t \\ \left(\sum_{j=0}^{\infty} C_j L^j \right) \left(I_n - \sum_{j=1}^{p} B_j L^j \right) &= I_n \end{aligned} \qquad \text{(A.2)}$$

We write this out as:

$$(C_0 + C_1 L + C_2 L^2 + \ldots)\,(\mathrm{I}_n - B_1 L - B_2 L^2 - \ldots) = \mathrm{I}_n \qquad \text{(A.3)}$$

and collect terms on the left-hand side:

$$C_0 + (C_1 - C_0 B_1)L + (C_2 - C_1 B_1 - C_0 B_2)L^2 + \ldots = \mathrm{I}_n \qquad \text{(A.4)}$$

The generic j-th element of this series is equal to:

$$\sum_{j=0}^{\infty} \left(C_j - \sum_{k=1}^{j} C_{j-k} B_k \right) L^j = I_n \qquad \text{(A.5)}$$

We now equate coefficients on powers of L on both sides of the equation. In order to do so, first note that $C_0 = \mathrm{I}_n$ and the rest of the C_j follow recursively, i.e., knowledge of the individual B_j matrices allows us to recover each of the C_j matrices required for the impulse response function:

$$C_1 - C_0 B_1 = C_1 - B_1 = 0 \quad \text{or} \quad C_1 = B_1$$

$$C_2 - C_1 B_1 - C_0 B_2 = 0 \quad \text{or} \quad C_2 = C_1 B_1 + B_2 = B_1^2 + B_2$$

$$C_3 - C_2 B_1 - C_1 B_2 - C_0 B_3 = 0 \quad \text{or} \quad C_3 = B_1^3 + 2B_1 B_2 + B_3$$

…

and the k-th element, C_k, can be computed recursively as:

$$C_k = \sum_{j=1}^{k} C_{k-j} B_j \tag{A.6}$$

Just to reiterate, any stable (or covariance stationary) VAR(p) process has a Wold (moving-average) representation of the form:

$$y_t - \mu = \varepsilon_t + C_1\varepsilon_{t-1} + C_2\varepsilon_{t-2} + \ldots = \sum_{j=0}^{\infty} C_j \varepsilon_{t-j} = C(L)\varepsilon_t \tag{A.7}$$

where $C_0 = \mathrm{I}_n$ and $\mu = \mathrm{E}(Y_t)$.

The C_i matrices can be interpreted as the dynamic multipliers of the system since they represent the model's response to a unit shock in each of the variables. The (i,j)-th element of:

$$C_k = \frac{\partial Y_t}{\partial \varepsilon_{t-k}} = \frac{\partial Y_{t+k}}{\partial \varepsilon_t} = c_{ij,k} \quad i, j = 1, \ldots, n \tag{A.8}$$

is denoted by $c_{ij,k}$ and represents the impact of a shock hitting the j-th variable of the system at time t on the i-th variable of the system at time t+k.

The sequences $c_{ij,k}$ (regarded as a function of k) are known as the **impulse response function**:

- $c_{ij,0}$ = initial impact (equal to one);
- $c_{ij,1}$ = impact after one period;
- $c_{ij,2}$ = impact after two periods;
- and so on…

So the impulse response function is given by:

$$\{\mathrm{I}_n, C_1, C_2, \ldots\}$$

Sometimes we are interested in **accumulated** effects, i.e., the overall effect after n periods:

$$C_n = \sum_{j=0}^{n} C_j \tag{A.9}$$

We also have a **long-run** effect (also called the total effect or total multiplier):

$$C_\infty = \sum_{j=0}^{\infty} C_j = C(1) = (I - C_1 - \ldots - C_p)^{-1} \tag{A.10}$$