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Economic modelling and forecasting

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Factor-augmented VARs (FAVARs)

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What do we know?

Large amounts of data about the state of the economy and the rest of the world…are collected, processed and analysed before each major [monetary policy] decision

L E O Svensson (2005, p. 4)

Most empirical analyses of monetary policy have been confined to frameworks in which the Federal Reserve is implicitly assumed to exploit only a limited amount of information, despite the fact that the Fed actively monitors literally thousands of economic time series

Bernanke and Boivin (2003, p. 525)

Outline

- The basic VAR model and the curse of dimensionality
- Drawbacks of (S)VAR models
- Monetary policy in SVARs
- Factor-augmented VARs:
  - rationale for factor models;
  - what are the factors;
  - estimation of factors;
  - illustration of Bernanke *et al.* (2005); and
  - illustration of Belviso and Milani's (2006) structural FAVAR
- Other benefits of using FAVAR models
- Summary

The need for new econometric techniques

- Especially when forecasting, the researcher wants to include as much information as possible, and it can be desirable to work with as many variables as possible…
- …but this can lead to models with a large number of variables and a proliferation of parameters
- New methods for estimating and forecasting given time-series observations for a large data set, which at time t is denoted by the N-dimensional vector X_t, are therefore required
- The need for new methods arises from the fact that, it is either **inefficient** or downright **impossible** to incorporate X_t in a single forecasting model and estimate it using standard econometric techniques

Case study: the basic VAR(p) model (1)

- A VAR consists of $t = 1, ... , T$ observations on a set of M endogenous macroeconomic variables $Y_t = (y_{1t}, \dots , y_{Mt})'$, such that Y_t is a $(M \times 1)$ vector containing T observations on M time series
- The VAR(p) process with p lags is then defined as:

$$Y_t = c + A_1 Y_{t-1} + \dots + A_p Y_{t-p} + \varepsilon_t \qquad (1)$$

where:

- the A_i are $(M \times M)$ coefficient matrices for $i = 1, \dots , p$;
- c is a $(M \times 1)$ vector of intercepts; and
- $\varepsilon_t = (\varepsilon_{1t}, \dots , \varepsilon_{Mt})'$ is an unobservable, $(M \times 1)$ error vector with $E(\varepsilon_t) = 0$ and time-invariant (i.e., constant), positive-definite variance-covariance matrix $cov(\varepsilon_t) = E(\varepsilon_t \varepsilon_t') = \Sigma$ (white noise), such that $\varepsilon_t \sim N(0, \Sigma)$

Case study: the basic VAR(*p*) model (2)

- Why the focus on VARs?
- To begin with, VARs are a standard framework for studying the effects of shocks on macroeconomic variables
- Moreover, atheoretical VAR models frequently serve as a benchmark against which the implications of theoretical models can be compared:
  - the empirical success of theoretical models is then assessed on the basis of how well the theoretical model's impulse response functions can approximate those derived from an (atheoretical) VAR

Problems with the basic VAR(*p*) model

- The number of coefficients in the basic (reduced-form) VAR(*p*) model in (1) easily proliferates, meaning that:
  - there are lots of coefficients to estimate – the total number of parameters to be estimated equals $M(Mp + 1)$;
  - unrestricted OLS estimates of the coefficient values are often not very well determined (in the sense of imprecise) in a finite set of data; and
  - if the $A_1, \ldots, A_p$ are imprecisely estimated because of limited data, the impulse response functions, forecasts and forecast error variance decompositions that are based on them will also be imprecisely estimated
- Alternative methods for estimating the coefficients have therefore been developed (…with a common theme being '**shrinkage**')

Problems with a small-scale (S)VAR model (1)

- Consider the following monetary SVAR(1) with GDP, inflation and the interest rate, such that $Y_t = (y_t, \pi_t, R_t)'$:

$$\underbrace{\begin{pmatrix} y_t \\ \pi_t \\ R_t \end{pmatrix}}_{Y_t} = \underbrace{\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}}_{c} + \underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}}_{A_1} \underbrace{\begin{pmatrix} y_{t-1} \\ \pi_{t-1} \\ R_{t-1} \end{pmatrix}}_{Y_{t-1}} + \underbrace{\begin{pmatrix} a_y & 0 & 0 \\ b_y & b_\pi & 0 \\ c_y & c_\pi & c_R \end{pmatrix}^{-1}}_{D^{-1}} \underbrace{\begin{pmatrix} u_{yt} \\ u_{\pi t} \\ u_{Rt} \end{pmatrix}}_{u_t} \qquad (2)$$

  where $D^{-1}u_t = \varepsilon_t$

- This specification implies that u_{Rt} (and, hence, R_t) has zero impact on y_t and π_t contemporaneously

Problems with a small-scale (S)VAR model (2)

- Multiplying through by *D* results in:

$$\underbrace{\begin{pmatrix} a_y & 0 & 0 \\ b_y & b_\pi & 0 \\ c_y & c_\pi & c_R \end{pmatrix}}_{D}\underbrace{\begin{pmatrix} y_t \\ \pi_t \\ R_t \end{pmatrix}}_{Y_t} = \underbrace{\begin{pmatrix} a_y & 0 & 0 \\ b_y & b_\pi & 0 \\ c_y & c_\pi & c_R \end{pmatrix}}_{D}\underbrace{\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}}_{c}$$

$$+\underbrace{\begin{pmatrix} a_y & 0 & 0 \\ b_y & b_\pi & 0 \\ c_y & c_\pi & c_R \end{pmatrix}}_{D}\underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}}_{A_1}\underbrace{\begin{pmatrix} y_{t-1} \\ \pi_{t-1} \\ R_{t-1} \end{pmatrix}}_{Y_{t-1}} \qquad (3)$$

$$+\underbrace{\begin{pmatrix} a_y & 0 & 0 \\ b_y & b_\pi & 0 \\ c_y & c_\pi & c_R \end{pmatrix}}_{D}\underbrace{\begin{pmatrix} a_y & 0 & 0 \\ b_y & b_\pi & 0 \\ c_y & c_\pi & c_R \end{pmatrix}^{-1}}_{D^{-1}}\underbrace{\begin{pmatrix} u_{yt} \\ u_{\pi t} \\ u_{Rt} \end{pmatrix}}_{u_t}$$

Problems with a small-scale (S)VAR model (3)

- Multiplying through by D results in the structural VAR:

$$\underbrace{\begin{pmatrix} a_y & 0 & 0 \\ b_y & b_\pi & 0 \\ c_y & c_\pi & c_R \end{pmatrix}}_{D} \underbrace{\begin{pmatrix} y_t \\ \pi_t \\ R_t \end{pmatrix}}_{Y_t} = \underbrace{\begin{pmatrix} c_1^* \\ c_2^* \\ c_3^* \end{pmatrix}}_{c^*} + \underbrace{\begin{pmatrix} a_{11}^* & a_{12}^* & a_{13}^* \\ a_{21}^* & a_{22}^* & a_{23}^* \\ a_{31}^* & a_{32}^* & a_{33}^* \end{pmatrix}}_{A_1^*} \underbrace{\begin{pmatrix} y_{t-1} \\ \pi_{t-1} \\ R_{t-1} \end{pmatrix}}_{Y_{t-1}} + \underbrace{\begin{pmatrix} u_{yt} \\ u_{\pi t} \\ u_{Rt} \end{pmatrix}}_{u_t} \quad (4)$$

Problems with a small-scale (S)VAR model (4)

- Putting everything together, we get:

$$
\begin{aligned}
a_y y_t &= c_1^* + a_{11}^* y_{t-1} + a_{12}^* \pi_{t-1} + a_{13}^* R_{t-1} + u_{yt} \\
b_\pi \pi_t &= -b_y y_t + c_2^* + a_{11}^* y_{t-1} + a_{12}^* \pi_{t-1} + a_{13}^* R_{t-1} + u_{\pi t} \\
c_R R_t &= -c_y y_t - c_\pi \pi_t + c_3^* + a_{11}^* y_{t-1} + a_{12}^* \pi_{t-1} + a_{13}^* R_{t-1} + u_{Rt}
\end{aligned}
\qquad (5)
$$

- The equation for R_t in (5) is similar to a Taylor rule: it implies that u_{yt} (i.e., y_t) and $u_{\pi t}$ (i.e., π_t) have a contemporaneous impact on R_t
- Hence it follows that the central bank sets interest rates by looking at fluctuations in y_t and π_t **only** – how realistic is that?

Drawbacks of SVARs (1)

- (S)VARs are widely used to assess the effect of (monetary policy) innovations on the economy…
- …and generally deliver empirically plausible assessments of the dynamic responses of key macroeconomic variables to monetary policy innovations
- But the empirical results depend on a sparse set of variables included in the model: standard VARs rarely employ more than six to eight variables to save degrees of freedom
- Such a small number of variables is:
  - liable to an omitted variable problem;
  - liable to measurement error; and
  - unlikely to span the entire information set used by central banks

Monetary policy in SVARs

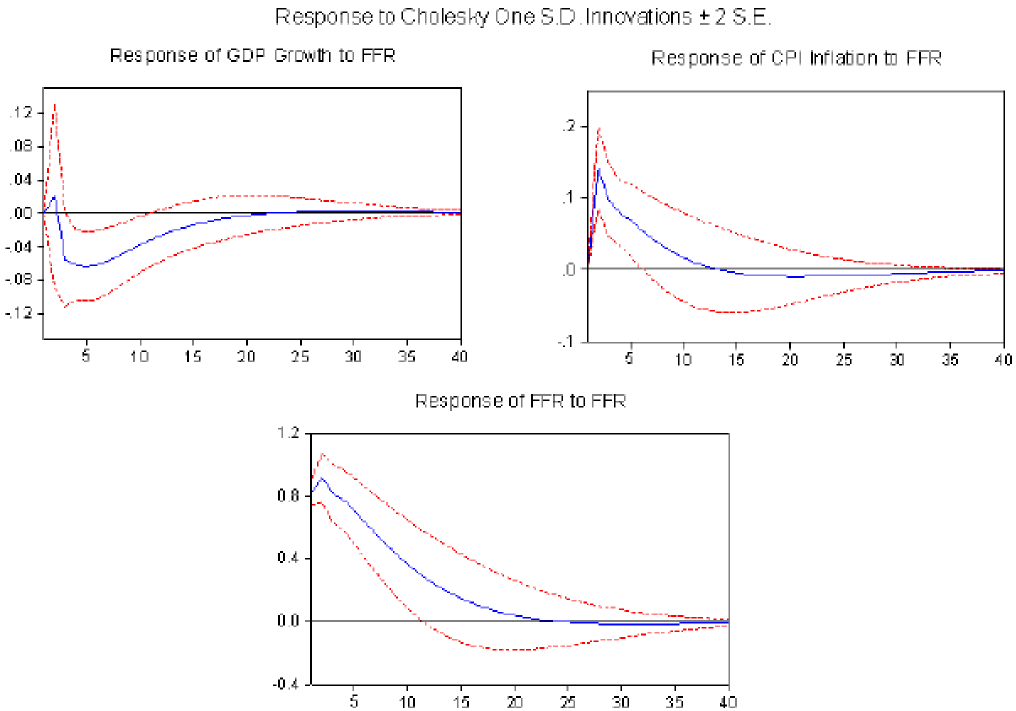
- Typical closed-economy benchmark SVAR models of (US) monetary policy consist of six variables (Christiano *et al.* (1999), Leeper *et al.* (1996), Strongin (1995)):
  - GDP, CPI, the commodity price level, the short-term interest rate (as the instrument of monetary policy), the quantity of total bank reserves and the amount of non-borrowed reserves

  and include some non-recursiveness in the system
- Models are generally identified by estimating a money supply equation, a money demand equation as well as the monetary policy reaction function in the process

Drawbacks of SVARs (2)

- There are additional problems with (S)VAR analyses of the effect of monetary policy innovations on the economy:
  - the choice of a specific variable to represent a general theoretic economic concept such as ‘real’ activity is often arbitrary to some degree (and also prone to measurement error);
  - to the extent that central banks and the private sector have information beyond that included in the SVAR, the measurement of (monetary) policy innovations (shocks) is likely to be contaminated and most likely incorrect; and
  - impulse responses can only be observed for the included variables, which generally constitute only a small subset of the variables that the central bank cares about

Consequences of mis-specified (S)VARs



- Mis-specified VARs throw up several well-documented puzzles, the most prominent being the **price puzzle**:
  - inflation **increases** after a monetary contraction

The FAVAR approach

- Bernanke *et al.* (2005) argue that these problems can be mitigated by adding more data to the VAR model
- But is it possible to condition VAR analyses of monetary policy on richer information sets without giving up the statistical advantages of restricting the analysis to a small number of series?
- Recent research in factor analysis suggests that the information from a large number of time series can be usefully summarised by a relatively small set of **estimated** indexes (?) or factors (Stock and Watson (2011)):
  - combine standard (structural) VAR analysis with factor analysis for large data sets

How do factor models work?

- Factor models exploit the fact that there may be a source of common fluctuations in a vector of economic time series
- The approach assumes that a low-dimensional vector of unobservable variables (the so-called ‘factors’) drives the co-movements across variables in the vector of economic time series
- The factors can therefore be considered as an exhaustive summary of the information contained in a (large) data set

What are these factors?

- Not only do these unobserved factors provide a better assessment of ‘real’ or ‘nominal’ conditions in the economy than any particular single variable…
- …but they may indeed capture fluctuations in unobserved potential output or reflect theoretically motivated concepts such as ‘real economic activity’, ‘price pressures’ or ‘credit conditions’ that cannot easily be represented by one or two series but rather are reflected in a wide range of economic variables

Set-up of the FAVAR (1)

- If a small number of estimated factors effectively summarise large amounts of information about the economy, then a natural solution to the curse of dimensionality or the degrees-of-freedom problem is to augment a standard VAR (consisting of an $(M \times 1)$ vector Y_t of observable economic variables) with (a 'small' $(K \times 1)$ vector of) these unobserved factors, F_t
- This approach, first suggested by Bernanke *et al.* (2005), is known as the **factor-augmented VAR** (**FAVAR**)...
- ...and consists of two equations

Set-up of the FAVAR (2)

- The $(M \times 1)$ vector, Y_t, of observable economic variables and the 'small' $(K \times 1)$ vector of unobserved factors, F_t, are combined in a **transition equation**:

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + v_t \qquad (6)$$

  where $\Phi(L)$ is a conformable lag polynomial of finite order d (which may contain *a priori* restrictions as in the SVAR literature) and v_t is an error term with mean zero and covariance matrix Σ

- Setting $Z_t = (F_t, Y_t)'$, (6) can be identified as a VAR(d), where:

$$Z_t = \Phi(L) Z_{t-1} + v_t$$

$$= c + \sum_{i=1}^{d} A_i Z_{t-i} + v_t = c + A_1 Z_{t-1} + \ldots + A_d Z_{t-d} + v_t \qquad (6')$$

FAVARs and VARs are related (1)

- The system in (6) reduces to a standard VAR in Y_t if the terms of $\Phi(L)$ that relate Y_t to $F_{t\text{-}1}$, $\phi_{21}(L)$, are all zero:

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \begin{bmatrix} \varphi_{11}(L) & \varphi_{12}(L) \\ \varphi_{21}(L) & \varphi_{22}(L) \end{bmatrix} \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + \begin{bmatrix} v_t^F \\ v_t^Y \end{bmatrix}$$

$$F_{\mathrm{t}} = \phi_{11}(L)F_{t\text{-}1} + \phi_{12}(L)Y_{t\text{-}1} + v_t^F \qquad (7)$$

$$\begin{aligned} Y_{\mathrm{t}} &= \phi_{21}(L)F_{t\text{-}1} + \phi_{22}(L)Y_{t\text{-}1} + v_t^Y \\ &= \phi_{22}(L)Y_{t\text{-}1} + v_t^Y \qquad (8) \end{aligned}$$

FAVARs and VARs are related (2)

- Because the FAVAR model nests standard VAR analyses, we can easily compare results with existing VARs (using a likelihood-ratio test, say) and assess the marginal contribution of the additional information contained in F_t
- When the true model is a FAVAR, estimation of a standard VAR in Y_t – that is, with the factors, F_t, omitted – will in general lead to biased estimates of the VAR coefficients and the related quantities of interest, such as impulse responses

Set-up of the FAVAR (3)

- But (6) cannot be estimated directly because the factors, F_t, are unobservable (state) variables that have to be estimated
- We therefore introduce a **second** equation
- Suppose that we have a 'large' set of N informational or background (zero-mean, stationary) time series, X_t, where $K + M \ll N$, which is related to the unobservable factors, F_t, and – sometimes, but not always – the observable variables, Y_t, by an **observation equation**:

$$X_t = \Lambda(F_t, Y_t) + e_t = \Lambda^f F_t + \Lambda^y Y_t + e_t \qquad (9)$$

where the Λ^i ($i = f, y$) are the factor loadings

- X_t is a ($T \times N$) panel data matrix which contains a range of macroeconomic and financial variables – the individual X_{it} need to be (approximately) **stationary** and, possibly, **standardised**

More on the factors (1)

- As an aside, note that Stock and Watson (2002a,b) refer to (9) – without the observable variables, Y_t – as a **dynamic factor model** (DFM):

$$X_t = \Lambda(F_t) + e_t = \Lambda^f F_t + e_t \qquad (10)$$

- The key aspect of the factor model is the introduction of F_t, which is a $(K \times 1)$ vector of observed latent factors which contains information extracted from all M variables
- The factors are common to every dependent variable (i.e., the same F_t occurs in every equation for X_{it} for $i = 1, \ldots, M$), but they may have different coefficients, Λ^f
- In other words, the researcher gets the benefit of using all N variables in X_t by using only the K factors, F_t, where K is typically much smaller than N

More on the factors (2)

- In principle, the factors should be estimated using an unobserved components model and the Kalman filter
- But, as argued by Stock and Watson (2009), the factors are still estimated consistently by principal components even with certain types of breaks or time variation in the loading parameters
- The choice of which data to include in X_t might not be innocuous
- While in theory more data are always better (Stock and Watson (2002b)), in practice that often means more of the same type of data like, for instance, more measures of real activity
- Boivin and Ng (2006) find that factors extracted from as few as 40 (pre-screened) series often yield satisfactory or even better results than using a much larger informational set, X_t

Estimation of factors (1)

- Estimation of factors can proceed in two ways:
  - either a two-step (semi-parametric) method, in which the factors, F_t, are estimated by principal components prior to the estimation of the factor-augmented VAR given by (6); or a
  - (fully parametric) one-step method, which makes use of Bayesian likelihood methods and Gibbs sampling to estimate the factors, F_t, and the dynamics simultaneously in a state-space model
- These approaches differ in various dimensions and it is not clear *a priori* that one should be favoured over the other
- Bernanke *et al.* (2005) address this issue and find that while the two methods produce qualitatively similar results, the two-step approach tends to produce more 'plausible' responses (incidentally, the same results holds for the structural FAVAR in Belviso and Milani (2006))

Estimation of factors (2)

- The time series for the first factor (principal component) is generated as a linear combination of the variables contained in X_t, with weights equal to the eigenvector associated with the largest eigenvalue
- By construction, the first factor (principal component) explains the greatest proportion of the variance of the dataset
- The remaining (orthogonal) factors (principal components) are calculated accordingly – with each one of them explaining a decreasing proportion of the variance of the dataset
- Note that these are purely statistical constructs…
- …and that we do not necessarily have an economic interpretation of the estimated principal components (factors)

Estimation of factors (3)

- Even though the two-step approach implies the presence of generated regressors in the second step…
- …the key result in this context is that the principal components estimator of the space spanned by the factors is consistent and, moreover, if N is sufficiently large (and the number of principal components is at least as large as the true number of factors), then the factors are estimated precisely enough to be treated as data in subsequent regressions (Stock and Watson (2011))

Empirical example: Bernanke *et al.* (2005) (1)

- Bernanke *et al.* (2005) illustrate their FAVAR methodology and underline the importance of considering a long list of information variables in the context of a simplified Rudebusch and Svensson (1999) model of monetary policy
- Their set of informational time series, X_t, consists of a balanced panel of 120 monthly (stationary) US macroeconomic time series

Empirical example: Bernanke *et al.* (2005) (2)

- The division of variables between F_t and Y_t in (6) depends on which variable(s) are assumed to be directly observed:
  - the authors assume that the most realistic description of the information structure is that the central bank observes only the policy instrument (the nominal interest rate), i.e., $Y_t = FFR_t$, as well as a large set of noisy macroeconomic indicators, X_t
- But it is likely that the nominal interest rate (the federal funds rate, or FFR_t) is contemporaneously related to some subset of variables in X_t, in particular the financial variables, commonly denoted as '**fast-moving**' variables

Rotating the factors

- In the Bernanke case with $Z_t = (F_t, FFR_t)'$, the principal components recover the space spanned by F_t and FFR_t
- That is, some of the recovered factors, F_t, could capture the role of FFR_t, which is contemporaneously correlated with some of the fast-moving variables
- We remove this correlation using the following steps (a process known as rotating the factors):
  - estimate F_t as above using the principal components estimator;
  - estimate $F_{slow,t}$ using only variables in X_t that are slow-moving (macroeconomic) variables;
  - calculate $F_{new,t} = F_t - B \times FFR_t$, where B is the coefficient on FFR_t in the regression $F_t = \alpha + D \times F_{slow,t} + B \times FFR_t + \varepsilon_t$; and then
  - estimate the FAVAR(p) in (6) using $F_{new,t}$ and FFR_t

Estimating the FAVAR

- The second step therefore involves estimating the FAVAR(*d*) in (6) by OLS:

$$Z_t = c + \sum_{i=1}^{d} A_i Z_{t-i} + v_t$$

  where $Z_t = (F_{new,t}, FFR_t)'$
- The monetary policy shock is identified using a Cholesky decomposition with the ordering $F_{new,t}$, FFR_t, meaning that FFR_t does not affect $F_{new,t}$ contemporaneously (note that this is only possible **after** rotating the principal components/factors)
- We can then calculate the impulse responses of $F_{new,t}$ and FFR_t to the policy shock

A two-step FAVAR estimator: summary

- **Step 1**: Approximate F_t as K principal components of X_t, where X_t is stationary (and, possibly, standardised)
- **Step 2**: Rotate the principal components to obtain $F_{new,t}$
- **Step 3**: Estimate a VAR using $F_{new,t}$ and FFR_t and estimate the impulse response to a monetary policy shock using the standard Cholesky decomposition
- **Step 4**: Recover the impulse responses of X_t and the units of the impulse responses
- (**Step 5**: Calculate the impulse response standard errors using the bootstrap procedure based on Kilian (1998), say)

The number of principal components

- Stock and Watson (2002a,b, 2005) suggest some guidelines for determining the number of factors
- The number of factors can be determined by a combination of *a priori* knowledge, visual inspection of a scree plot and the use of information criteria developed by Bai and Ng (2002):
  - we can use a (subjective) measure such as the proportion of variance explained;
  - we can use formal statistics like the appropriately defined information criteria due to Bai and Ng (2002) for the number of factors present in the data set, X_t; or
  - Bernanke *et al.* (2005) increase K until there is no change in the impulse response functions − they find that the first three principal components do a good job in summarising the information in their dataset

Structural FAVAR (SFAVAR) models (1)

- While FAVARs enjoy a number of advantages over standard VARs:
  - they allow for a better identification of the monetary policy shock;
  - they avoid the use of a single variable to proxy theoretical constructs; and
  - they allow researchers to compute impulse response functions for all the variables in the underlying dataset

  they do have a shortcoming, namely that the factors are not (uniquely) identified and lack an economic interpretation
- For that reason, Belviso and Milani (2006) propose a structural FAVAR (SFAVAR) model, in which the factors have a clear(er) meaning

Structural FAVAR (SFAVAR) models (2)

- In particular, they propose seven structural factors:
  - real activity;
  - inflation (or price pressures);
  - interest rates;
  - financial markets;
  - money;
  - credit (or credit conditions); and
  - expectations
- Where do these factors come from?
- The starting point is again the ‘large’ set of *N* informational or background (zero-mean, stationary) time series, X_t

Structural FAVAR (SFAVAR) models (3)

- We then take a partition of X_t, say, X_t^1, X_t^2, … , X_t^I, where X_t^i (i = 1, … , I) is a ($N_i \times 1$) vector and $\Sigma_i N_i = N$
- Assume that each of the vectors X_t^i (i = 1, … , I) is explained by only some of the elements of the vector, F_t
- That is, there is a partition of F_t given by F_t^1, F_t^2, … , F_t^I, where F_t^i (i = 1, … , I) is a ($K_i \times 1$) vector and $\Sigma_i K_i = K$ and $K_i < N_i$ for all i
- Also, assume that X_t^i is only explained by F_t^i, which results in:

$$\begin{bmatrix} X_t^1 \\ X_t^2 \\ \vdots \\ X_t^I \end{bmatrix} = \begin{bmatrix} \Lambda_1^f & 0 & \cdots & 0 \\ 0 & \Lambda_2^f & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Lambda_I^f \end{bmatrix} \begin{bmatrix} F_t^1 \\ F_t^2 \\ \vdots \\ F_t^I \end{bmatrix} + \begin{bmatrix} e_t^1 \\ e_t^2 \\ \\ e_t^I \end{bmatrix} \qquad (11)$$

Structural FAVAR (SFAVAR) models (4)

- The restriction the authors impose on the model is that each of the variables in the X_t^i vector (i = 1, …, I) is influenced by the state of the economy only through the corresponding i-th factor
- The authors also assume that each segment of X_t is explained by exactly one factor, that is $K_i = 1$ for all i
- Indeed, the main contribution of Belviso and Milani (2006) is the set of restrictions illustrated in (11)

Structural FAVAR (SFAVAR) models (5)

- As before, the dynamics of (F_t^1, … , F_t^l, Y_t) are given by a FAVAR(d) of finite lag order d:

$$\begin{bmatrix} F_t^1 \\ F_t^2 \\ \vdots \\ F_t^l \\ Y_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1}^1 \\ F_{t-1}^2 \\ \vdots \\ F_{t-1}^l \\ Y_{t-1} \end{bmatrix} + v_t \qquad (12)$$

where Φ(L) is a conformable lag polynomial of finite order d and v_t is an error term with mean zero and covariance matrix Σ

Structural FAVAR (SFAVAR) models (6)

- Assume that the vector of economic variables, X_t, is divided into subsets of similar variables, for example, a subset of variables related to real activity, a subset of variables related to inflation, etc.
- This means that the dynamic factors, i.e., the common forces that move these economic variables, are now economically interpretable:
  - these forces represent more general concepts such as economic activity, basic movements in prices and so forth

Structural FAVAR (SFAVAR) models (7)

- Returning to the seven structural factors, these are:
  - **real activity**: industrial production, capacity utilisation, employment/unemployment indicators, inventory stocks, new and unfilled orders, consumer expenditures, etc.;
  - **inflation (or price pressures)**: consumer prices, retail prices, producer prices, wages, oil prices, etc.;
  - **interest rates**: interest rates at different medium- and long-term maturities on both public and private fixed-income instruments;
  - **financial markets**: asset prices;
  - **money**: money stock variables, deposits, bank reserves, etc.;
  - **credit (or credit conditions)**: private credit and loan variables, etc.; and
  - **expectations**: production, employment, inventories, new orders, inflation, short-term interest rates

Structural FAVAR (SFAVAR) models (8)

- Now that we have derived factors that are economically interpretable, we can examine their reaction – and the reaction of the variables used in their construction – to a monetary policy shock
- Following convention, the system is identified using a Cholesky decomposition, which implies a recursive ordering of the variables
- One problem arising from the system is the presence of the **interest rate factor**, which includes data on several longer-term interest rates
- Allowing the policy rate, Y_t, to respond to the interest rate factor leads to an identification problem: we would run the risk of confusing an arbitrage condition with the policy rule

Structural FAVAR (SFAVAR) models (9)

- In the Cholesky ordering, the interest rates as well as the expectations factor are therefore ordered **after** the monetary policy instrument, Y_t
- In other words, we allow for a contemporaneous response of Y_t to the other factors: inflation, real activity, credit, money and financial markets (which react to monetary policy only with a lag)
- We retain one of the advantages of the FAVAR framework, namely that we can derive impulse-response functions not only for the (structural) factors but also for all the variables explained by the factors

Extensions of (S)FAVAR models

- Identification of the structural shocks can be achieved by methods other than short-run restrictions using the Cholesky decomposition: Ahmadi and Uhlig (2012) for example, employ sign restrictions for this purpose
- Moreover, Banerjee and Marcellino (2009) have extended the FAVAR model to the factor-augmented error-correction model, which makes it possible to include variables that are non-stationary

Other benefits of using FAVAR models

- Stock and Watson (1999) and Bernanke and Boivin (2003) have shown that, generally, factor methods are useful for forecasting inflation
- Banerjee *et al.* (2006) show that dynamic factor models are a reasonable alternative forecasting tool in the face of short spans of reliable time series:
  - despite the constraints on the time-span of data, a large number of macroeconomic series of potential use in forecasting (for a given time span) are available

Summary (1)

- VARs generally provide a credible approach to capturing the time-series properties of a vector time series as well as forecasting and structural inference
- But the number of coefficients in a VAR easily proliferates, putting limits on the number of variables that can be included in the VAR and on the precision of coefficient estimates
- One approach to addressing this curse of dimensionality is the factor-augmented VAR (FAVAR)

Summary (2)

- The FAVAR combines standard VAR analysis with factor analysis to exploit large datasets in the study of monetary policy
- FAVARs augments a standard VAR with a ‘small’ vector of estimated factors
- Under realistic assumptions about the information structure (i.e., are all variables in the model assumed to be directly observed?), the implied empirical model is more likely to be a FAVAR than a standard VAR
- Factor analysis can be used to incorporate a broad range of conditioning information, summarised by a small number of factors, into an otherwise standard SVAR analysis

Summary (3)

- By employing as much information as possible in the construction of the forecasting exercise, factor models provide a methodology that allows us to remain ‘agnostic’ about the (rapidly-changing?) structure of the economy
- The number of variables in the information or background data set, X_t, does not need to be extremely large for the principal components estimator to give reasonably precise estimates
- It is also possible to do structural analysis in FAVAR models (an example is provided in, *inter alia*, Belviso and Milani (2006))

References and further reading (1)

Bai, J and Ng, S (2002), ‘Determining the number of factors in approximate factor models’, *Econometrica*, Vol. 70, No. 1, pages 191-221.

Bai, J and Ng, S (2006), ‘Confidence intervals for diffusion index forecasts and inference for factor-augmented regressions’, *Econometrica*, Vol. 74, No. 4, pages 1133-50.

Banerjee, A and Marcellino, M (2009), ‘Factor-augmented error correction models’, Chapter 9 in Castle, J L and Shephard, N (eds), *The methodology and practice of econometrics: a festschrift in honour of David F Hendry*, Oxford University Press, Oxford, pages 227-54.

References and further reading (2)

Banerjee, A, Marcellino, M and Masten, I (2006), 'Forecasting macroeconomic variables for the new member states', Chapter 4 in Artis, M, Banerjee, A and Marcellino, M (eds), *The central and eastern European countries and the European Union*, Cambridge, Cambridge University Press. http://www.eui.eu/Personal/Marcellino/22.pdf.

Barhoumi, K, Darné, O and Ferrara, L (2014), 'Dynamic factor models: a review of the literature', *Journal of Business Cycle Measurement and Analysis*, Vol. 2013, No. 2, pages 73-107.

Belviso, F and Milani, F (2006), 'Structural factor-augmented VARs (SFAVARs) and the effects of monetary policy', *Topics in Macroeconomics*, Vol. 6, No. 3, Article 2. http://www.bepress.com/bejm/topics/vol6/iss3/art2/.

References and further reading (3)

Bernanke, B S and Boivin, J (2003), ‘Monetary policy in a data-rich environment’, *Journal of Monetary Economics*, Vol. 50, No. 3, pages 525-46.

Bernanke, B S, Boivin, J and Eliasz, P (2005), ‘Measuring the effects of monetary policy: a factor-augmented vector autoregressive (FAVAR) approach’, *Quarterly Journal of Economics*, Vol. 120, No. 1, pages 387-422.

Boivin, J and Ng, S (2006), ‘Are more data always better for factor analysis?’, *Journal of Econometrics*, Vol. 132, No. 12, pages 169-94.

References and further reading (4)

Boivin, J, Kiley, M T and Mishkin, F S (2010), 'How has the monetary transmission mechanism evolved over time?, Chapter 8 in Friedman, B M and Woodford, M (eds), *Handbook of monetary economics, Vol 3*, Amsterdam, Elsevier, pages 369-422. http://www.federalreserve.gov/pubs/feds/2010/201026/201026pap.pdf.

Christiano, L J, Eichenbaum, M and Evans, C L (1999), 'Monetary policy shocks: what have we learned and to what end?', Chapter 2 in Taylor, J B and Woodford, M (eds), *Handbook of Macroeconomics, Vol.1, Part A*, Amsterdam, Elsevier, pages 65-148.

References and further reading (5)

Eklund, J and Kapetanios, G (2008), 'A review of forecasting techniques for large data sets', *National Institute Economic Review*, Vol. 203, No. 1, pages 109-15.

Eickmeier, S, Lemke, W and Marcellino, M (2011), 'Classical time-varying FAVAR models – estimation, forecasting and structural analysis', *Deutsche Bundesbank Discussion Paper No 04/2011, Series 1: Economic Studies*. http://www.bundesbank.de/download/volkswirtschaft/dkp/2011/201104dkp.pdf.

Kilian, L (1998), 'Small-sample confidence intervals for impulse response functions', *Review of Economics and Statistics*, Vol. 80, No. 2, pages 218-30.

References and further reading (6)

Leeper, E M, Sims, C A and Zha, T (1996), 'What does monetary policy do?', in Brainard, W C and Perry, G L (eds), *Brookings Papers on Economic Activity*, Vol. 1996, No. 2, pages 1-78 (with comments and discussion). http://www.brookings.edu/~/media/Projects/BPEA/1996%202/1996b_bpea_leeper_sims_zha_hall_bernanke.PDF.

Ludvigson, S C and Ng, S (2007), 'The empirical risk-return relation: a factor analysis approach', *Journal of Financial Economics*, Vol. 83, No. 1, pages 171-222.

Ludvigson, S C and Ng, S (2009), 'Macro factors in bond risk premia', *Review of Financial Studies*, Vol. 22, No. 12, pages 5027-67.

References and further reading (7)

Stock, J H and Watson, M W (1999), 'Forecasting inflation', *Journal of Monetary Economics*, Vol. 44, No. 2, pages 293-335.

Stock, J H and Watson, M W (2002a), 'Forecasting using principal components from a large number of predictors', *Journal of the American Statistical Association*, Vol. 97, No. 460, pages 1167-79.

Stock, J H and Watson, M W (2002b), 'Macroeconomic forecasting using diffusion indexes', *Journal of Business & Economic Statistics*, Vol. 20, No. 2, pages 147-62.

Stock, J H and Watson, M W (2005), 'Implications of dynamic factor models for VAR analysis', *NBER Working Paper No. 11467*. http://www.nber.org/papers/w11467.pdf.

References and further reading (8)

Stock, J H and Watson, M W (2009), 'Forecasting in dynamic factor models subject to structural instability', Chapter 7 in Castle, J L and Shephard, N (eds), *The methodology and practice of econometrics: a festschrift in honour of David F Hendry*, Oxford, Oxford University Press, pages 173-205.

Stock, J H and Watson, M W (2011), 'Dynamic factor models', Chapter 2 in Clements, M P and Hendry, D F (eds), *The Oxford handbook of economic forecasting*, Oxford, Oxford University Press, pages 35-60. https://www.princeton.edu/~mwatson/papers/dfm_oup_4.pdf.

Stock, J H and Watson, M W (2012), 'Generalised shrinkage methods for forecasting using many predictors', *Journal of Business & Economic Statistics*, Vol. 30, No. 4, pages 481-93.

References and further reading (9)

Strongin, S (1995), 'The identification of monetary policy disturbances: explaining the liquidity puzzle', *Journal of Monetary Economics*, Vol. 35, No. 3, pages 463-97.

Svensson, L E O (2005), 'Monetary policy with judgment: forecast targeting', *International Journal of Central Banking*, Vol. 1, No. 1, pages 1-54. http://www.ijcb.org/journal/ijcb05q2a1.pdf.