Recent developments in structural VAR modelling

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Recent developments in structural VAR modelling

Outline

- Motivating structural VARs
- Structural and reduced-form VARs
- VAR and SVAR representations
- The identification problem
- Identifying structural shocks:
  - long- and short-run zero (exclusion) restrictions
  - sign restrictions
  - alternative SVAR identification approaches
- Which shocks are driving which variable:
  - historical decompositions
- Summary
Main applications of structural VARs

- Structural VAR models have four main applications:
  - they are used to study the expected response of the model variables to a given one-time **structural** or macroeconomic shock (impulse response functions or IRFs);
  - they allow the construction of forecast error variance decompositions (FEVDs) that quantify the average contribution of a given **structural** or macroeconomic shock to the variability of the data;
  - they can be used to provide historical decompositions that measure the cumulative contribution of each variable (shock) over time; and
  - they allow for the construction of forecast scenarios conditional on hypothetical sequences of future **structural** or macroeconomic shocks
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From a reduced-form VAR to a structural VAR (1)

• SVARs have become one of the major ways of extracting information about the macroeconomy, i.e., structural analysis

• To determine this information a reduced-form VAR is first fitted to summarise the data and then a structural VAR is proposed whose structural equation errors are taken to be the structural, primitive or economic shocks or innovations

• The parameters of these structural equations are then ‘identified’ by utilising the information in the estimated reduced-form VAR

• The VAR is a reduced-form model which summarises the data, while the SVAR provides an interpretation of the data
VAR and SVAR representations (1)

• Assume that the data contained in the \((n \times 1)\) vector of variables \(y_t\) can be represented by a vector autoregression (VAR) with \(p\) lags, denoted VAR(\(p\))
• For simplicity, we assume the following VAR(1) model:
  \[ y_t = A_1 y_{t-1} + \varepsilon_t \]  \hspace{1cm} (1)
where \(\varepsilon_t\) is a set of errors that have zero expectation, constant covariance matrix, \(\Omega\), and no serial correlation
• The VAR given by (1) summarises the data
For an interpretation of the data, we need to use the structural VAR (SVAR) given by:

\[ B_0 y_t = B_1 y_{t-1} + u_t \]  

(2)

where the \( u_t \) are shocks that have zero mean, no serial correlation, constant variance-covariance matrix \( \Sigma \) and no correlation between the individual shocks, i.e., \( \text{E}[u_{it}u_{jt}] = 0 \).

Comparing (1) and (2), we find that:

\[ B_0 \varepsilon_t = u_t \]  

(3)

such that the structural shocks, \( u_t \), we seek to measure are linear combinations of the observed VAR errors, \( \varepsilon_t \).

The latter can be estimated by the VAR residuals, \( \hat{\varepsilon}_t \).
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VAR and SVAR representations (3)

- Typically, the matrix $\Sigma$ is specified as a **diagonal** matrix because the structural shocks are assumed to originate from independent sources.
- They are hence purely **exogenous** and **mutually uncorrelated**.
- Furthermore, the variance-covariance matrix of the structural error term, $u_t$, is frequently normalised such that:
  \[ \text{cov}(u_t) = \text{E}(u_t u_t') = \Sigma = I_n \]  
  \hspace{1cm} (4)
- In summary, the assumptions underlying structural shocks are that:
  - there are as many structural shocks as variables in the model;
  - structural shocks by definition are mutually uncorrelated, which implies that $\Sigma$ is diagonal; and
  - we can normalise the variances of all structural shocks to unity.
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The identification problem (1)

- What information do we have to recover the structural shocks?
- Compare the coefficients between (1) and (2):
  \[ A_1 = B_0^{-1} B_1 \]
  \[ \varepsilon_t = B_0^{-1} u_t \quad \text{or} \quad u_t = B_0 \varepsilon_t \]
- We also know the variance-covariance matrix of residuals, \( \mathbb{E}(\varepsilon_t \varepsilon_t') = \Omega \) (...or at least we can estimate it from the data)
- Any candidate matrix \( B_0 \) linking the reduced-form shocks with the structural shocks must satisfy:
  \[
  \Omega = \mathbb{E}(\varepsilon_t \varepsilon_t') = \mathbb{E}(B_0^{-1} u_t (B_0^{-1} u_t)') \\
  = \mathbb{E}(B_0^{-1} u_t u_t' (B_0^{-1})') = B_0^{-1} \mathbb{E}(u_t u_t')(B_0^{-1})' \\
  = B_0^{-1} \Sigma(B_0^{-1})' 
  \] (5)
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The identification problem (2)

- We generally assume that the structural shocks, $u_t$, are uncorrelated and have unit variance (this is a normalisation), which means that $\text{cov}(u_t) = E(u_t u'_t) = \Sigma = I_n$

- Condition (5) then becomes:

\[
\begin{align*}
\Omega &= E(\varepsilon_t \varepsilon'_t) = E(B_0^{-1}u_t(B_0^{-1}u'_t)) \\
&= E(B_0^{-1}u_t u'_t(B_0^{-1})) = B_0^{-1}E(u_t u'_t)(B_0^{-1})' \\
&= B_0^{-1}\Sigma(B_0^{-1})' = B_0^{-1}I_n(B_0^{-1})' \\
&= B_0^{-1}(B_0^{-1})' \quad (6)
\end{align*}
\]

- Mechanically, identification therefore means finding a matrix $B_0$ that satisfies:

\[
\Omega = B_0^{-1}(B_0^{-1})' \quad (7)
\]

- One such matrix is the Cholesky decomposition of $\Omega$, offering one possible solution to the problem of how to recover $u_t$.
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**VAR and SVAR representations (4)**

- The solution to the VAR(1) is the following infinite moving average (MA) form:
  \[ y_t = D_0 \varepsilon_t + D_1 \varepsilon_{t-1} + D_2 \varepsilon_{t-2} + \ldots \]  \( (8) \)
  where \( D_j \) is the \( j \)-th period impulse response of \( y_{t+j} \) to a unit change in \( \varepsilon_t \) (\( D_0 = I_n \))

- It follows that the infinite MA form for the SVAR(1) is:
  \[ y_t = D_0 \varepsilon_t + D_1 \varepsilon_{t-1} + D_2 \varepsilon_{t-2} + \ldots = D_0 B_0^{-1} u_t + D_1 B_0^{-1} u_{t-1} + D_2 B_0^{-1} u_{t-2} + \]
  \[ = C_0 u_t + C_1 u_{t-1} + C_2 u_{t-2} + \ldots \]  \( (9) \)
  with the \( j \)-th period impulse response of \( y_{t+j} \) to \( u_t \) being \( C_j = D_j B_0^{-1} = D_j C_0 \) as \( C_0 = B_0^{-1} \)
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From a reduced-form VAR to a structural VAR (2)

• As for any set of structural equations, identifying the structural shocks requires the use of identification restrictions that reduce the number of ‘free’ parameters in the structural equations to the number that can be recovered from the information in the reduced form

• There are several major methods in the applied literature:
  – the Cowles Commission approach;
  – employing a recursive causal structure;
  – (non-recursive) short-run restrictions;
  – long-run restrictions;
  – long- and short-run restrictions;
  – sign restrictions; and
  – other methods
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**Long- and short-run restrictions**

- We can, of course, also combine different sorts of restrictions:
  - it is the overall number of restrictions that counts, not their nature
- For example, Bjørnland and Leitemo (2009) have combined long- and short-run restrictions...
- ...while Dungey and Pagan (2000) have proposed a hybrid approach, under which (zero) exclusion restrictions are placed on the structural dynamics (the $B_i$'s) as well as $B_0$
Identification by sign restrictions (1)

- As has been pointed out by Sarno and Thornton (2004), there is a fundamental problem related to the identification of macroeconomic shocks to financial variables in SVARs:
  - under the assumption of efficient markets (i.e., variables reflect all information relevant for their determination), the zero covariance restrictions that are typically employed in an SVAR identification are inappropriate; and
  - they may have to be replaced with alternative restrictions, such as sign restrictions (Mönch (2012))
- While placing a single efficient-market variable last in a recursive ordering overcomes the above problem, it need not be the correct solution
- And what do we do if the VAR includes two or more financial market variables?
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Identification by sign restrictions (2)

- Scepticism toward traditional identifying assumptions based on either short-run or long-run restrictions has made an alternative class of SVAR models more popular.
- Starting with Faust (1998), structural shocks in these SVAR models are identified by restricting the sign of the responses of selected model variables to structural shocks.
- Uhlig (2005) showed that sign-identified models may produce substantially different results from conventionally identified SVAR models.
Identification by sign restrictions (3)

- Identification in sign-identified models requires that each identified shock is associated with a unique sign pattern.
- Sign restrictions may be static, in which case we simply restrict the sign of the coefficients in $B_0^{-1} (B_0)$.
- Unlike traditional exclusion restrictions, such sign restrictions can often be motivated directly from economic theory.
- There is a misperception among many users that sign-identified models are more general and hence more credible than VAR models based on exclusion restrictions – that is not the case!
- By construction, sign-identified models are more restrictive than standard VAR models in some dimensions and less restrictive in others.
- But they do not nest models based on exclusion restrictions.
Sign restrictions in practice (1)

- Estimate the reduced-form VAR coefficients and compute the residual variance-covariance matrix, Ω.
- Let $u_t$ denote the corresponding SVAR model innovations.
- The construction of the structural impulse response function requires an estimate an the $(n \times n)$ matrix $B_0^{-1}$ in $\varepsilon_t = B_0^{-1}u_t$.
- Using the Cholesky decomposition, we find one structural form, $C_0$, that satisfies $\Omega = C_0C_0'$.
- Then $B_0^{-1} = C_0R$ also satisfies $\Omega = B_0^{-1}(B_0^{-1})'$ for any orthogonal $(n \times n)$ matrix $R$ (i.e., $R' = R^{-1}$ or $RR' = RR = I_n$):

\[
B_0^{-1} = C_0R \implies C_0 = B_0^{-1}R^{-1}
\]
\[
\Omega = C_0C_0' = B_0^{-1}R^{-1}((B_0^{-1})'(R^{-1})') = B_0^{-1}R^{-1}(R^{-1})'(B_0^{-1})' = B_0^{-1}I_n(B_0^{-1})' = B_0^{-1}(B_0^{-1})'
\] (10)
Sign restrictions in practice (2)

• Unlike $C_0$, $C_0R$ will in general be **non-recursive**
• One can examine a wide range of possible solutions $B_0^{-1} = C_0R$ by repeatedly drawing at random from the (uniformly distributed) set $R$ of orthogonal (rotation) matrices
• For each draw, rotate the Cholesky decomposition and compute the structural IRFs
• Following Rubio-Ramirez *et al.* (2010), one constructs the set of admissible models by drawing from the set $R$ and discarding candidate solutions for $B_0^{-1}$ that do not satisfy a set of *a priori* sign restrictions on the implied impulse response functions
• Keep doing this until you accept $N$ draws
Sign restrictions in practice (3)

• More specifically, the procedure consists of the following steps:
  – draw a \((n \times n)\) matrix \(L\) of NID(0,1) random variables and derive the \(QR\) decomposition of \(L\) such that \(L = QR\) and \(QQ' = I_n\);
  – let \(T = Q'\) and compute impulse responses using the orthogonalisation \(B_0^{-1} = PT\) – if all implied impulse response functions satisfy the identifying restrictions, retain \(T\), otherwise discard \(T\); and
  – repeat the first two steps a large number of times, recording each \(T\) that satisfies the restrictions (and the corresponding impulse response functions)

• The resulting set \(B_0^{-1}\) in conjunction with the reduced-form estimates then characterises the set of admissible SVAR models
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Identification by sign restrictions: example (1)

• Consider a small macro model involving the output gap \((y_t)\), inflation \((\pi_t)\) and a policy interest rate \((R_t)\), i.e., \(z_t = (y_t, \pi_t, R_t)\)'

• A typical SVAR(1) model for these variables would be:

\[
\begin{align*}
y_t &= z'_{t-1} y + \alpha_{yR} R_t + \alpha_{y\pi} \pi_t + \varepsilon_{yt} \\
\pi_t &= z'_{t-1} \pi + \alpha_{\pi R} R_t + \alpha_{\pi y} y_t + \varepsilon_{\pi t} \\
R_t &= z'_{t-1} \pi + \alpha_{\pi Y} y_t + \alpha_{\pi R} R_t + \varepsilon_{Rt} 
\end{align*}
\]

(11) (12) (13)

• The three structural or economic shocks will be a demand shock \((\varepsilon_{yt})\), a cost-push (supply) shock \((\varepsilon_{\pi t})\) and a monetary policy shock \((\varepsilon_{Rt})\)
Identification by sign restrictions: example (2)

• The signs of the contemporaneous effects to positive shocks could be as follows:

<table>
<thead>
<tr>
<th>Variable/shock</th>
<th>Demand</th>
<th>Cost-push</th>
<th>Interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t$</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$R_t$</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

• Since the patterns of the sign restrictions are distinct, this suggests that we are likely to be able to identify separate shocks
• Indeed, shocks having a distinct sign pattern in their effects on variables is clearly a requirement if we are to isolate them separately
Identification by sign restrictions: example (3)

• The signs of the contemporaneous effects to positive shocks could be as follows:

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<td>+</td>
<td>–</td>
</tr>
<tr>
<td>$R_t$</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

• But we need to recognise the problem with different signs, as there will now be nine possible signs for $C_0$ that are compatible with the presence of the three postulated shocks.

• The need to check for the complete set of compatible sign restrictions will grow as the number of shocks increases.
Identification by sign restrictions (4)

- In addition, one may restrict the sign of impulse responses at longer horizons, although the theoretical rationale of such restrictions is usually weaker.
- Stemming mainly from the Monte Carlo work in Paustian (2007), there is a belief in the literature that adding sign restrictions for longer impulse responses, $C_j, j > 0$, provides stronger identifying information.
- But, as demonstrated in Fry and Pagan (2011), it is clear from the (recursive) connections that exist between the $C_j$ and $C_0$ (the initial impulse response) that nothing guarantees this (in particular, $C_j = D_jC_0$).
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Identification by sign restrictions (5)

• Note that the same reduced-form VAR model is used in the construction of all candidate models

• All candidate models therefore fit the data equally well by construction...

• ...and there is no way of evaluating the validity of the identifying restrictions based on the reduced form

• Note also that small fraction of admissible models is not an indication of how well (or not) the identifying restrictions fit the data...

• ...but the fraction of initial candidate models that satisfy the identifying restrictions may be viewed as an indication of how informative the identifying restrictions are about the structural parameters
Interpretation of sign restrictions (1)

- A fundamental problem in interpreting VAR models identified by sign restrictions is that there is not a unique point estimate of the structural impulse response functions.
- Unlike conventional SVAR models based on short-run or long-run restrictions, sign-identified VAR models are only set identified.
- This problem arises because sign restrictions represent inequality restrictions:
  - the cost of remaining agnostic about the precise values of the structural model parameters is that the data are potentially consistent with a wide range of structural models that are all admissible in that they satisfy the identifying restrictions.
- Without further assumptions there is no way of knowing which of these models is most likely.
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Interpretation of sign restrictions (2)

• What should we do about the multiple models problem?
• The standard approach has been to report a central tendency, such as the vector of pointwise medians of the structural impulse response functions, and the magnitude of the spread of responses
• This approach suffers from two distinct shortcomings:
  – the median response function lacks a structural economic interpretation (Fry and Pagan (2011)); and
  – median response functions are not a valid statistical summary of the set of admissible impulse response functions, as the vector of medians is not the median of vectors
• But it is important to recognise that the distribution is across models and has nothing to do with sampling uncertainty
Summary of sign restrictions

- Sign restrictions are a way to identify structural shocks that avoid the use of exclusion (zero) restrictions (and recursive structures).
- Sign restrictions can be motivated from economic theory.
- They will usually have larger error bands than the Cholesky decomposition, which is due to the fact that there are many rotation matrices that satisfy the sign restriction.
- In sum, there is a misperception among many users that these models are more general (agnostic) and hence more credible than VAR models based on exclusion restrictions...
- ...which is not the case, as forcefully argued in Fry and Pagan (2011).
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**Alternative SVAR identification approaches**

- Alternative identification approaches for SVAR models that have been suggested in the literature include:
  - financial market shocks (Faust et al. (2004), D’Amico and Farka (2011));
  - identification by heteroskedasticity (Rigobon (2003)); and
  - identification in the presence of forward-looking behaviour
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Identification by financial market shocks

• Faust et al. (2004) identify monetary policy shocks in monthly VAR models based on high-frequency futures market data.

• In particular, using the prices of daily federal funds futures contracts, they measure the impact of the surprise component of Federal Reserve policy decision on the expected future trajectory of interest rates.

• The procedure involves two key steps:
  – the futures market is used to measure the response of expected future interest rates to an unexpected change in the Federal Reserve’s target rate; and
  – the authors impose that the impulse response function of the federal funds rate to the monetary policy shock in the VAR model must match the response measured from the futures data.
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Identification by heteroskedasticity

• Rigobon (2004) develops a method for solving the VAR identification problem based on the heteroskedasticity of the structural shocks

• In the baseline model, heteroskedasticity can be described as a two-regime process and the structural parameters of the system are just identified

• The approach is of particular interest when modelling asset prices because instantaneous feedback must be assumed when trading is near-continuous, but not without serious limitations…

• …primarily about the existence, number and timing of variance regimes
Identification by forward-looking behaviour

• Standard VAR models of monetary policy are concerned with responses to unanticipated policy shocks…
• …meaning that have nothing to say about effects of anticipated monetary policy shocks (this is of even bigger concern when analysing fiscal policy shocks)
• The mere possibility of forward-looking behaviour greatly complicates the identification of structural shocks in VAR models
• The starting point is therefore a structural vector moving-average (SVMA) model
• There are no generic solutions to the problem of modelling forward-looking behaviour in VARs, but illustrative examples exist (Barsky and Sims (2011) and Leeper et al. (2013))
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**Historical decompositions in theory**

- The paper by Burbridge and Harrison (1985) investigated the extent to which monetary shocks contributed to the Great Depression...
- ...and was the first to use the concept of a **historical decomposition**:

\[
y_t = \sum_{i=0}^{\infty} C_i \epsilon_{t-i} \tag{14}
\]

\[
y_{T+j} = \sum_{i=0}^{j-1} C_i \epsilon_{T+j-i} + \sum_{i=j}^{\infty} C_i \epsilon_{T+j-i} \tag{15}
\]

- The time series under investigation is split into two parts:
  - the projected values based on some origin \( T \); and
  - the ‘news’ since time \( T \)
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Historical decompositions in practice (1)

- What is the contribution of different shocks to this news?
- Decide on the origin for the baseline projection, i.e., choose \( T \)
- Estimate the VAR and construct the baseline projection:
  \[
  Y_{T+j} = A^{j-i}Y_T
  \]  
  (16)
- Back out the structural shocks from the VAR residuals:
  \[
  u_t = D^{-1}\varepsilon_t
  \]  
  (17)
- Feed in one structural shock at a time for \( T + j \), with \( j > 0 \)…
- …and graph the outcome
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Historical decompositions in practice (2)

- Example: historical decomposition of US GDP
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Summary (1)

• With the help of structural VARs, the economic structure that determines any economic outcome is inferred from the observed data and a structural interpretation of the data is obtained from economic theory
• But there are alternative economic theories and, consequently, alternative structural interpretations of the same observations
• While the success of reduced-form VAR models as descriptive tools and – to some extent – as forecasting tools is well established...
• ...the ability of structural representations of VAR models to differentiate between correlation and causation, in contrast, has remained contentious
Summary (2)

- Specifying fully structural models of the macroeconomy in recursive form has been largely abandoned.
- VAR models identified by sign restrictions are the most popular alternative to VAR models identified by short-run or long-run exclusion restrictions.
- But in practical applications, it is often found that a combination of all the methods mentioned above need to be employed in order to be able to identify all the shocks of interest.
- Which of the above methods is used in practice does not depend on the data, but rather on the preferences of the investigator wishing to use SVARs to study some issue.
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References and further reading (1)


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References and further reading (2)


References and further reading (3)


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References and further reading (4)


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References and further reading (5)


http://pubs.aeaweb.org/mwg-internal/de5fs23hu73ds/progress?id=Bt6OeDlxkIa5C5nu61dBwgD3KDgukAkHV6e8MX1RnRE,&dl.


References and further reading (6)


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References and further reading (7)


Recent developments in structural VAR modelling

References and further reading (8)


References and further reading (9)


References and further reading (10)