BANK OF ENGLAND

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# An Intro to the New Keynesian DSGE Model

**Date**

Nairobi, April 28, 2016

**Title**

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# Introduction

- The New Keynesian model forms the basic building block of COMPASS
- It combines the micro fundamentals of the Real Business Cycle literature with nominal rigidities
- Two fundamental differences:
  - Imperfect competition coupled with nominal rigidities (adjustment costs to prices)
  - Monetary sector introduced, making it possible to investigate real effects of monetary policy
- Without rigidities in pricing, introducing money does not lead to monetary policy having real effects

# The effects of money: intuition

- Take the equation for the quantity of money

$$\frac{M}{P} = cY$$

- With flexible prices: $\uparrow M \Rightarrow P$, $Y$ unaffected
- With price rigidities: $\uparrow M \Rightarrow$ prices adjust partially $\Rightarrow \uparrow Y$ (money is not neutral in the short-run)

# Key agents in the model

HOUSEHOLDS

Maximise utility subject to the budget constraint

FIRMS

INTERMEDIATE GOODS PRODUCING FIRMS

Produce intermediate goods. Maximise profits. Market is imperfectly competitive, they set prices.

Intermediate Goods →

FINAL GOODS PRODUCING FIRMS

Assemble intermediate goods and sell them on to households. The market is perfectly competitive

GOVERNMENT

Issues bonds and makes money transfers

**THE DOMESTIC ECONOMY**

# A basic closed economy model

- More formally, the building blocks of a prototype New Keynesian model are:

  1. Households: consume differentiated goods, hold money, own firms, offer labor
  2. Firms: produce differentiated intermediate goods (they are price setters) and ‘bundle’ final products in perfectly competetive markets; all firms demand labor, pay dividends
  3. Government: make money transfer payments financed by the creation of money

# Representative household

- During each period $t = 0, 1, 2 \ldots,$ the representative household chooses $\{C_t, L_t, M_t/P_t, B_t/P_t\}$, to maximise

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} + a_m \ln\left(\frac{M_t}{P_t}\right) - \frac{1}{\eta} L_t^{\eta} \right] \right\},$$

- Subject to the budget constraint

$$\frac{M_{t-1} + B_{t-1} + T_t + W_t L_t + D_t}{P_t} = C_t + \frac{B_t/R_t + M_t}{P_t},$$

- Where $M$, $B$, $R$, and $W$ are nominal money, nominal bonds, gross interest rates and the nominal wage respectively; $D$, $L$ and $C$ are real profits, hours worked and total consumption

# Representative household

- To solve the problem of the representative household we can set up the Lagrangian function as:

$$\sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \left[ \frac{C_t^{1-\sigma}}{1-\sigma} + a_m \ln\left(\frac{M_t}{P_t}\right) - \frac{1}{\eta} L_t^{\eta} \right] \\ + \Lambda_t \left[ \frac{M_{t-1} + B_{t-1} + T_t + W_t L_t + D_t}{P_t} - C_t - \frac{B_t / R_t + M_t}{P_t} \right] \end{array} \right\}$$

- Where $\Lambda_t$ is the Lagrange multiplier on the budget constraint
- The representative household chooses $\{C_t, L_t, M_t/P_t, B_t/P_t\}$ to maximize the Lagrangian

# First order conditions

- The first order conditions for this problem are:

$$C_t \quad : \quad C_t^{-\sigma} = \Lambda_t \qquad (1)$$

$$L_t \quad : \quad \Lambda_t \frac{W_t}{P_t} = L_t^{\eta-1} \qquad (2)$$

$$B_t/P_t \quad : \quad \frac{\Lambda_t}{R_t P_t} = \beta E_t \frac{\Lambda_{t+1}}{P_{t+1}} \qquad (3)$$

$$M_t/P_t \quad : \quad a_m \left(\frac{M_t}{P_t}\right)^{-1} = \frac{\Lambda_t}{P_t} - \beta E_t \frac{\Lambda_{t+1}}{P_{t+1}} \qquad (4)$$

# First order conditions

- We can substitute equation (1) into (2) and (3) to write:

$$\frac{W_t}{P_t} = \frac{L_t^{\eta-1}}{C_t^{-\sigma}}$$
$$\frac{C_t^{-\sigma}}{R_t P_t} = \beta E_t \frac{C_{t+1}^{-\sigma}}{P_{t+1}}$$

- These two equations represent the labor supply and the Euler equation for consumption respectively
- We can substitute equations (1) and (3) into (4) to write:

$$\frac{M_t}{P_t} = \frac{R_t}{R_t - 1} \frac{a_m}{C_t^{-\sigma}}$$

- This equation is the LM equation

# Basic intuition

- FOC on $L_t$: Captures the consumption/leisure "trade-off"
  - The marginal rate of substitution between consumption and leisure must be equal to the real wage
- FOC on $B_t/P_t$: Euler equation
  - The marginal utility of today's consumption must be equal to the marginal utility of tomorrow's consumption in present discounted terms
- FOC on $M_t/P_t$: Money demand
  - At the margin, the cost (measured in terms of utility) of not consuming today must be equal to the benefit of holding another unit of money. Another unit of money yields utility and also allows future consumption

# Representative final goods producer

- The representative final goods producing firm uses $Y_t(i)$ units of each intermediate good $i$ to produce $Y_t$ according to the CES technology

$$\left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} = Y_t$$

- It sells at the nominal price $P_t$ in order to maximize its profits subject to the CES technology: hence, the problem of the representative goods producing firm is to choose $Y_t(i)$ to maximize

$$\max_{Y_t(i)} P_t \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_t(i) Y_t(i) di$$

# Representative final goods producer

- The FOC is

$$Y_t(i) = (P_t(i)/P_t)^{-\varepsilon} Y_t$$

- This represents the demand for intermediate goods that the representative final goods producing firm has: The parameter $\varepsilon$ is the elasticity of demand
- If $\uparrow P_t(i)$ then $\downarrow Y_t(i)$: Also, as $\varepsilon \to \infty$, individual goods become closer substitutes and individual firms have less power
- If we substitute this last expression into the CES production technology, after a bit of algebra, we obtain the price level

$$P_t = \left[ \int_0^1 P_t(i)^{\varepsilon-1} di \right]^{\frac{1}{\varepsilon-1}}$$

# Representative intermediate goods producer

- Demand for each intermediate firm's good is given by (solution to the previous slide's maximisation problem):

$$Y_t(i) = (P_t(i)/P_t)^{-\varepsilon} Y_t$$

- Note that this is a downward-sloping demand curve, $\varepsilon$ is the elasticity of demand
- They face this given demand function for their goods and must either choose the price or the quantity produced
- Since intermediate good producing firms are monopolistic competitors, they choose $P_t(i)$ to maximise profits subject to their demand function and a given production technology (to be defined)

# Representative intermediate goods producer

- During each period $t = 0, 1, 2 \ldots$, the representative intermediate goods producer hires $L_t(i)$ units of labor from the household to manufacture $Y_t(i)$ units of intermediate good $i$ according to the production technology

$$Y_t(i) = Z_t L_t(i)$$

- The aggregate technology shock, $Z_t$, follows the law of motion

$$\ln(Z_t/\overline{Z}) = \rho \ln(Z_{t-1}/\overline{Z}) + \xi_t$$

where $0 < \rho < 1$, and $\xi_t \sim N(0, \mu^2)$

# Representative intermediate goods producer

- As in Rotemberg (1982), the representative intermediate goods producer faces a quadratic cost of adjusting its nominal price between periods

$$\frac{\phi_p}{2}\left(\frac{P_t(i)}{\overline{\Pi} P_{t-1}(i)} - 1\right)^2 Y_t$$

- Hence, any change in price is costly for the firm
- With this formulation both upwards and downwards changes affect the decision of the firm
- This is Stiglitz's idea of the implicit cost of changing prices

# Representative intermediate goods producer

- Hence, the problem of the intermediate goods producer is to choose $P_t(i)$ to maximize its total market value

$$\sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{P_t} \left( P_t(i) Y_t(i) - W_t L_t(i) - \frac{\phi_p}{2} \left( \frac{P_t(i)}{\overline{\Pi} P_{t-1}(i)} - 1 \right)^2 P_t Y_t \right)$$

subject to the production technology

$$Y_t(i) = Z_t L_t(i)$$

and the demand for intermediate goods

$$Y_t(i) = (P_t(i)/P_t)^{-\varepsilon} Y_t$$

# Representative intermediate goods producer

- The first order condition for the representative intermediate goods producer is

$$
\begin{aligned}
0 \; = \; & (1-\varepsilon)\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} + \varepsilon\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon-1}\frac{W_t}{P_t Z_t} \\
& -\phi_p\left(\frac{P_t(i)}{\overline{\Pi}P_{t-1}(i)} - 1\right)\frac{P_t(i)}{\overline{\Pi}P_{t-1}(i)} \\
& +\beta\phi_p E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{P_{t+1}(i)}{\overline{\Pi}P_t(i)} - 1\right)\frac{P_{t+1}(i)}{\overline{\Pi}P_t(i)}\frac{P_t Y_{t+1}}{P_t(i) Y_t}\right]
\end{aligned}
$$

- This is the (non-approximated version of the) New Keynesian Phillips curve

# Representative intermediate goods producer

- To develop the intuition, suppose $\phi_p \to 0$, so that there are no nominal rigidities in the economy
- Then the Phillips curve collapses to

$$0 = (1 - \varepsilon) + \varepsilon \left( \frac{P_t(i)}{P_t} \right)^{-1} \frac{W_t}{P_t Z_t}$$

which can be re-written as

$$\frac{P_t(i)}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{W_t}{P_t Z_t}$$

that is the familiar condition that states that the price is set as a markup over marginal costs

# Government, and aggregate constraint

- Government budget constraint:

$$(M_t - M_{t-1}) = T_t$$

- The economy resource constraint can be derived from the representative household, intermediate goods producer budget, and government constraints:

$$Y_t = C_t + \frac{\phi_p}{2}\left(\frac{P_t(i)}{\overline{\Pi}P_{t-1}(i)} - 1\right)^2 Y_t$$

# Symmetric equilibrium

- In a symmetric equilibrium, all intermediate goods producers make identical decisions, so that $Y_t(i) = Y_t$, $L_t(i) = L_t$, $P_t(i) = P_t$, and $D_t(i) = D_t$, for all $t = 0, 1, 2 \ldots$.
- In addition, market-clearing conditions $B_t = B_{t-1} = 0$, and $M_t = M_{t-1} + T_t$ must hold for all $t = 0, 1, 2 \ldots$.
- After imposing these conditions we can derive the equilibrium of the model

# Symmetric equilibrium

- The model describes the behaviour of 8 variables $\{C_t, W_t, M_t/P_t, Z_t, L_t, \Pi_t, R_t, Y_t\}$ by

  1. IS curve $\frac{C_t^{-\sigma}}{R_t P_t} = \beta E_t \frac{C_{t+1}^{-\sigma}}{P_{t+1}}$
  2. Labor supply $\frac{W_t}{P_t} = \frac{L_t^{\eta-1}}{C_t^{-\sigma}}$
  3. LM curve $\frac{M_t}{P_t} = \frac{R_t}{R_t - 1} \frac{a_m}{C_t^{-\sigma}}$
  4. Taylor rule $\ln(R_t/\overline{R}) = \theta_y \ln(Y_t/\overline{Y}) + \theta_\pi \ln(\Pi_t/\overline{\Pi}) + \epsilon_t$

# Symmetric equilibrium (ctd.)

- And

5. Stochastic technology shock $\ln\left(\frac{Z_t}{\overline{Z}}\right) = \rho \ln\left(\frac{Z_{t-1}}{\overline{Z}}\right) + \xi_t$
6. Production technology $Y_t = Z_t L_t$
7. Phillips curve

$$
\begin{aligned}
0 \quad = \quad & (1-\varepsilon) + \varepsilon \frac{W_t}{P_t Z_t} - \phi_p \left(\frac{\Pi_t}{\overline{\Pi}} - 1\right) \frac{\Pi_t}{\overline{\Pi}} \\
& + \beta \phi_p E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} \left(\frac{\Pi_{t+1}}{\overline{\Pi}} - 1\right) \frac{\Pi_{t+1}}{\overline{\Pi}} \frac{Y_{t+1}}{Y_t} \right]
\end{aligned}
$$

8. Aggregate resource constraint

$$
Y_t = C_t + \frac{\phi_p}{2} \left( \frac{P_t(i)}{\overline{\Pi} P_{t-1}(i)} - 1 \right)^2 Y_t
$$

# A 'workhorse' monetary policy rule

- Many models closed using a Taylor rule

$$r_t = \gamma r_{t-1} + (1-\gamma)\left[\theta_\pi \pi_{t+i} + \theta_y y_t\right] \qquad (5)$$

- The original Taylor (1993) empirical proposal was for $\theta_\pi = 1.5$, $\theta_y = 0.5$, $\gamma = 0$ (no smoothing), $i = 0$ (contemporaneous feedback)
- Is the model microfounded if the policy rule is not?
- Should it be?

# Why Taylor rules?

- Become a cornerstone of modern monetary policy analysis
  - Simple to understand
  - Reflects a move to 'models without money': Interest rate becomes the policy instrument
- The restriction that $\theta_\pi > 1$ should hold has become enshrined as the 'Taylor Principle'
- Variations used to explain monetary policy across regimes
  - Smoothing introduced to make dynamics more realistic
  - Forecast inflation targeting ($i > 0$) intended to reflect policy aims: To bring inflation back to target in the future
  - Influential empirical papers: Clarida et al. (1998); Theory in Taylor (ed.) (1999) Monetary Policy Rules

# Why Taylor rules?

- Taylor suggests the following policy rule table

| | | $y_t$ | | |
|---|---|---|---|---|
| | | -2 | 0 | 2 |
| $\pi_t$ | 0 | .5 | 1 | 2 |
| | 2 | 3 | 4 | 5 |
| | 4 | 6 | 7 | 8 |
| | 6 | 9 | 10 | 11 |
| | 8 | 12 | 13 | 14 |

- Does this imply we can all go home?

# Why not Taylor rules?

- The universality of Taylor rules doesn't mean that they have desirable properties
- Svensson (2003) (amongst many others) argues that a lot of effort has gone into microfounding models and none at all into microfounding Taylor rules
- It may be that some variant of a Taylor rule is (nearly) optimal
- It may be that there are many better simple alternatives
  - Long literature on simplicity and robustness
  - These use rules that don't rely on the model but instead on 'control principles'

# Welfare

- What microfoundations do we have for optimal policy?
- Woodford (2003) Interest and Prices: Write aggregate one-period utility

$$U(C_t, L_t) = u(Y_t; \xi_t) - \int_0^1 v(Y_t^j; \xi_t) dj$$

where $u(Y_t; \xi_t)$ is the utility of aggregate output

$Y_t \equiv \left[ \int_0^1 \left( Y_t^j \right)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$ and $v(Y_t^j; \xi_t)$ the disutility of supplying $Y_t^j$

- Uses solution for output $Y_t = Y^0 + B\epsilon_t$
- $Y^0$, $B$ are coefficients potentially affected by policy

# Welfare

- Turns out we can derive a quadratic approximation to welfare
- After much algebra Woodford shows that

$$U(C_t, L_t) \simeq -\frac{1}{2} u_c \overline{Y} \left( \mu_1 y_t^2 + \mu_2 \text{var}_j \left[ \log P_t^j \right] \right)$$

  with $\mu_1$, $\mu_2$ functions of the structural coefficients and $\text{var}_j \left[ \log P_t^j \right]$ the variance of log prices across all differentiated goods; this reflects price dispersion
- Pricing frictions will determine $\text{var}_j \left[ \log P_t^j \right]$
  - No frictions $\Rightarrow$ no variation $\Rightarrow$ no price dispersion: Term disappears
  - Calvo or Rotemberg pricing can be used to obtain the NK Philips curve

# Welfare

- Look to approximate welfare with a quadratic function of macro variables
- Woodford shows that for Calvo pricing

$$W_0 = E_0\left[\sum_{t=0}^{\infty}\beta^t U(C_t, L_t)\right]$$
$$\simeq E_0\left[-\Phi\sum_{t=0}^{\infty}\beta^t\left(\pi_t^2 + \omega y_t^2\right)\right] \quad (6)$$

where $\omega = \frac{\kappa}{\theta}$ and we omit a **lot** more algebra

- Objective function reduces down to an ‘old-fashioned’ ad hoc concern for output and inflation stabilisation

# Optimal policy

- We have derived an objective function that reflects welfare: Both policymakers and economic agents can act optimally
- Two important problems
  - Approximation depends on the form of pricing and the rest of the model: More complicated models make it much more difficult to derive even approximate welfare so (6) is often used
  - Optimal policy is inherently time inconsistent
- The first of these means that we may not know what we are trying to optimise; the second means that we cannot implement the policy anyway
- Time inconsistency is pervasive, but need not be fatal

# Characterising the optimal policy

- Form the Hamiltonian
$$H_t = \frac{1}{2}\left(\pi_t^2 + \omega y_t^2\right) + \lambda_t \left(\beta E_t \pi_{t+1} + \kappa y_t + z_t - \pi_t\right)$$
where $\lambda_t$ is a Lagrange multiplier
- We only need constrain the objective function using the Philips curve as we can always find a value of $i_t$ to satisfy the IS equation
- Using this we can write the constrained objective function as
$$V_0 = \min E_0 \sum_{t=0}^{\infty} \beta^t H_t \qquad (7)$$

# Characterising the optimal policy

- Write (7) explicitly as

$$
\begin{aligned}
V_0 \quad = \quad & \min \frac{1}{2}\left(\pi_0^2 + \omega y_0^2\right) + \lambda_0\left(\beta\pi_1 + \kappa y_0 + z_0 - \pi_0\right) \\
& + E_0\beta\left[\frac{1}{2}\left(\pi_1^2 + \omega y_1^2\right) + \lambda_1\left(\beta\pi_2 + \kappa y_1 - \pi_1\right)\right] \\
& + E_0\beta^2\left[\frac{1}{2}\left(\pi_2^2 + \omega y_2^2\right) + \lambda_2\left(\beta\pi_3 + \kappa y_2 - \pi_2\right)\right] \\
& + E_0\beta^3\left[\frac{1}{2}\left(\pi_3^2 + \omega y_3^2\right) + \lambda_2\left(\beta\pi_4 + \kappa y_3 - \pi_3\right)\right] + \dots
\end{aligned}
$$

- $\pi_t$ appears in two different constraints except for $\pi_0$
- Initial period is therefore different

# Characterising the optimal policy

- First order conditions

$$\begin{aligned}
\frac{\partial V_0}{\partial y_t} &= 0 \Rightarrow E_0\left(\omega y_t + \kappa \lambda_t\right) = 0 \\
\frac{\partial V_0}{\partial \pi_{t>0}} &= 0 \Rightarrow E_0\left(\pi_t - \lambda_t + \lambda_{t-1}\right) = 0 \\
\frac{\partial V_0}{\partial \pi_0} &= 0 \Rightarrow \left(\pi_0 - \lambda_0\right) = 0
\end{aligned}$$

# Characterising the optimal policy

- Solution for the Lagrange multiplier, $\lambda$

$$\lambda_t = -\frac{\omega}{\kappa} y_t$$

- Eliminating $\lambda$

$$\begin{aligned} \pi_{t>0} &= \Delta\lambda_t \\ &= -\frac{\omega}{\kappa}\Delta y_t \qquad (8) \\ \pi_0 &= \lambda_0 \\ &= -\frac{\omega}{\kappa} y_0 \qquad (9) \end{aligned}$$

- Yields a 'two part' policy for the policymaker to implement

# Policy under discretion

- The optimal policy relies on commitment
  - To sustain the optimal policy the monetary authority must have the reputation to stick to an announced plan
  - The optimal policy is treated as a rule, but we know it may not be the best policy in the future
- What if the policymaker retains discretion?
  - If policy is not set by an unbreakable rule then it can change policy in any given period
  - Think of successive governments who are not responsible for their predecessors actions
- From the perspective of period 0 this means discretionary policy can be no better and may be substantially worse

# Characterising discretionary policy

- Find the discretionary policy by setting $E_0 \left( E_t \pi_{t+1} \right) = \overline{\pi} = 0$
- Now write (7) as

$$
\begin{aligned}
V_0 \quad = \quad & \min \frac{1}{2} \left( \pi_0^2 + \omega y_0^2 \right) + \lambda_0 \left( \kappa y_0 + z_0 - \pi_0 \right) \\
& + E_0 \beta \left[ \frac{1}{2} \left( \pi_1^2 + \omega y_1^2 \right) + \lambda_1 \left( \kappa y_1 - \pi_1 \right) \right] \\
& + E_0 \beta^2 \left[ \frac{1}{2} \left( \pi_2^2 + \omega y_2^2 \right) + \lambda_2 \left( \kappa y_2 - \pi_2 \right) \right] \\
& + E_0 \beta^3 \left[ \frac{1}{2} \left( \pi_3^2 + \omega y_3^2 \right) + \lambda_2 \left( \kappa y_3 - \pi_3 \right) \right] + \ldots
\end{aligned}
$$

- $\pi_t$ now appears in only one ‘row’

# Characterising discretionary policy

- First order conditions are now

$$\frac{\partial V_0}{\partial y_t} = 0 \Rightarrow E_0 \left( \omega y_t + \kappa \lambda_t \right) = 0$$

$$\frac{\partial V_0}{\partial \pi_t} = 0 \Rightarrow E_0 \left( \pi_t - \lambda_t \right) = 0$$

- So the optimal discretionary policy is

$$\pi_t = \lambda_t = -\frac{\omega}{\kappa} y_t$$

- Cannot be time inconsistent

# Implications of time inconsistency

- If policymakers are forced to adopt time consistent policies then they may be substantially inferior
- In static models easy to show that there is an inflation bias
- In dynamic models there is a stabilisation bias — takes longer to deal with shocks
- Both can be reduced by having a policymaker who is more conservative than socially optimal but acts under discretion ($\omega_m < \omega$)
- Can show that a policymaker who 'smoothes' policy under discretion is also better (Woodford, 2003)

# Instrument rules and targeting rules

- Taylor rule (5) an obvious example of an instrument rule
  - Indicates how much the policy instrument should be moved to achieve a given target
- Svensson calls (8) and (9) a targeting rule
  - Indicates what policy should achieve without necessarily being explicit about how
- McCallum argues that operationally some de facto rule would need to be used
- See also McCallum and Nelson (2005); Svensson (2005)

# Concluding remarks

- We set up a prototype New Keynesian model from first principles
- Advantages of the framework: ‘micro-fundamentals’, ‘realistic’ monopolistic competition, role for stabilising monetary policy, inflation dynamics based on future expectations of marginal costs
- Simple policy rules may have very good operating characteristics
- Even if policymakers act to maximise welfare their announced policy is time inconsistent
- Discretionary policy may be very suboptimal

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