EXERCISE ACCOMPANYING THE SESSION:
AN INTRODUCTION TO DSGE MODELS

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Exercise 1. Asset Prices in a Heterogenous Agent Economy. Consider the following 2-period ($t \in \{0, 1\}$), single-good economy with two possible states in period 1, and three different agents. All agents have the same expected utility function given by

$$\log\left(c_0^i\right) + \beta \sum_{n \in \{1,2\}} \pi_n \log\left(c_n^i\right)$$

where $\beta = 0.99$ and the probabilities of the respective states are $\pi_1 = 0.6$ and $\pi_2 = 0.4$. Further assume the following endowment pattern:

i	y_0^i	y_1^i	y_2^i
1	2	5	2
2	6	6	3
3	1	4	6

- Use the representative agent result to quickly compute Arrow security prices Q_1 and Q_2.
- What would be the price and rate of return on a riskless bond (i.e. one paying a unit of the consumption good with certainty, in period 1) traded in period 0?
- Who is the richest agent in the model?
- Compute the consumption patterns of all agents (you can use Excel or Matlab).
- Verify whether the markets for consumption in every period clear. What are the implications for Arrow security markets?
- Comment on Agent 1's pattern of trade: which Arrow securities does he buy, which ones does he sell?
- Compute the ratio of consumption in state 1 to state 2 for all agents. Comment.

Exercise 2. The Deterministic Steady State of a DSGE Model. Find the steady state in the simple three-equation model:

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} (1 + R_t)$$

$$C_t = Y_t$$

$$\log(Y_t) = \rho \log(Y_{t-1}) + \varepsilon_t^Y$$

What is the numerical value of R when $\beta = 0.99$? Does it make sense?

Hint: You may have heard of the **Deterministic Steady State** (indeed before solving DSGE models, many pieces of software explicitly require you to specify one).

The steady state is a combination of values for all the *variables* in the model, such that

- *if* stochastic shocks were absent / equalled 0;
- and *if* all model variables were equal to their steady- state values in period $t-1$,
- *then* they would remain at these values also in period $t, t+1, t+2, \ldots$

Somewhat imprecisely, the steady state is where we would expect a model to 'settle' in the 'long run'.

To find the SS use the crucial elements of the definition:

- *if* stochastic shocks were absent:
  - which means that we can set all shocks to 0 (only when finding the SS, of course)
  - it also means that we can then omit the expectation operator
- *if* all model variables were equal to their steady state values in period $t-1$
- *then* they would remain at these values also in period $t, t+1, t+2, \ldots$
  - this means that, when solving for the SS, we can assume that all variables in the model are constant
  - we shall denote the constant steady state values of variables by omitting their time subscripts t, $t+1$ - e.g. C, Y, R etc.

Exercise 3. The log-linearised DSGE Model. Derive a log-linear approximation to the simple three-equation model around the steady state; can you guess why we need logs?

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} (1 + R_t)$$

$$C_t = Y_t$$

$$\log(Y_t) = \rho \log(Y_{t-1}) + \varepsilon_t^Y.$$