BANK OF ENGLAND

Centre for Central Banking Studies

An Introduction to DSGE Models

Date

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Title

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DSGE models

- First things first...
  - D - Dynamic
  - S - Stochastic
  - G - General
  - E - Equilibrium

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- By the end of these two sessions you should:
  - Be able to solve simple heterogenous agent DSGE models
  - Have an understanding of some of the key underlying concepts
    - Complete vs incomplete markets
    - Heterogeneous vs representative agent models
    - Links between utility specifications and model outcomes
    - Consumption-Euler equation
- Tomorrow: basic properties of the NK model

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The basic approach

- Clarify costs and benefits of actions
  - Done formally in an optimisation problem
- Standard (and familiar) example: how does a household divide income between consumption and saving
- History provides examples of interesting solutions (expectations matter!)...
- History suggests that accounting for how people respond to changes can be crucial for policymakers!

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A static deterministic general equilibrium model

- Initially, we shall keep things simple and solve a model which is
  - Static: i.e. there will be only one time-period, $t \equiv 1$
  - Deterministic: i.e. everything will be known at the time of making the decision
  - General Equilibrium: i.e. no agent will be able to improve their situation by unilaterally changing their behaviour
- To make things a little bit harder, we will consider a multiple good, heterogeneous agent model
  - I.e. there will be many goods traded and we will allow for differences between consumers
- Assumption:
  - Every household aims to attain the highest possible utility
  - Jargon: agent = consumer = household

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Utility

- We will denote consumer i's consumption of good n by c_n^i where $i \in I$ and $n \in \{0, \ldots, N\}$
- Need to be specific about agent i's utility function
- We have many different functional forms to choose from
- More broadly, we can have
- Key distinction between variables and parameters

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  - quadratic: $u(c_0^i, c_1^i, \ldots, c_N^i) = \gamma_0^i \left(c_0^i\right)^2 + \ldots +$
  - log: $u(c_0^i, c_1^i, \ldots, c_N^i) = \gamma_0^i \log\left(c_0^i\right) + \ldots +$
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  - separable utility: $u(c_0^i, c_1^i, \ldots, c_N^i) = f_0\left(c_0^i\right) + f_1\left(c_1^i\right) + \ldots + f_N\left(c_N^i\right)$
  - non-separable utility: any utility function which is not separable
    - E.g. $u(c_0^i, c_1^i) = c_0^i \cdot c_1^i$
- Key distinction between variables and parameters

Notes on utility

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  - No way of measuring utility
  - Different choices of utility functions could potentially lead to very different conclusions
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- Consumer $i \in I$ decides on consumption of $N+1$ goods to maximise utility

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2. Differentiate $\mathcal{L}(X, Y, \lambda)$ w.r.t. X, Y and λ and equate to 0

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   These equations are called the first-order conditions (FOCs)

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- The Lagrange multiplier approach to finding a solution

1. Define the Lagrangian $\mathcal{L}(X, Y, \lambda)$ as

$$\mathcal{L}(X, Y, \lambda) \equiv U(X, Y) - \lambda(PX + QY - B)$$

   where λ is called a Lagrange multiplier

2. Differentiate $\mathcal{L}(X, Y, \lambda)$ w.r.t. X, Y and λ and equate to 0

$$\frac{\partial \mathcal{L}}{\partial X} = \frac{\partial U}{\partial X} - \lambda P = 0 \quad \Leftrightarrow \quad \mathcal{L}_x = U_x - \lambda P = 0$$

$$\frac{\partial \mathcal{L}}{\partial Y} = \frac{\partial U}{\partial Y} - \lambda Q = 0 \quad \Leftrightarrow \quad \mathcal{L}_y = U_y - \lambda Q = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = PX + QY - B = 0 \quad \Leftrightarrow \quad \mathcal{L}_\lambda = PX + QY - B = 0$$

   These equations are called the first-order conditions (FOCs)

3. Use the equations to solve for X and Y. For us, they imply

$$\frac{U_x}{U_y} = \frac{P}{Q} \Leftrightarrow U_x Q - U_y P = 0$$

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Lagrange multipliers: a simple example

- To ensure that we understand how the technique of Lagrange multipliers works, let's apply it to a specific example:
  - Find the maximum of $U(X, Y) = XY + 2X$ subject to the constraint $4X + 2Y = 60$
- Solution: $\{X, Y\} = \{8, 14\}$

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Solving agent i's optimisation problem

- We can now apply Lagrange multipliers to the optimisation problem solved by consumer i

$$\max_{c_0^i, c_1^i, \ldots, c_N^i} \left\{ \gamma_0 \log\left(c_0^i\right) + \gamma_1 \log\left(c_1^i\right) + \ldots + \gamma_N \log\left(c_N^i\right) \right\}$$

$$s.t. \quad : \quad \sum_{n \in \{0,1,\ldots,N\}} p_n c_n^i = \sum_{n \in \{0,1,\ldots,N\}} p_n y_n^i$$

- What is consumer i's optimum expenditure on the consumption of good n?

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Individual excess demand / supply schedules - solution

- The desired expenditure on good n by consumer i is given by

$$\forall n \in \{0, \ldots, N\} : p_n c_n^i = \frac{\gamma_n}{\sum_{m \in \{0,1,\ldots,N\}} \gamma_m} \left(\sum_{m \in \{0,1,\ldots,N\}} p_m y_m^i \right)$$

  - What determines whether agent i buys/sells good n in the market?

- How does the quantity consumed depend on the price of good n?
  - What is the intuition behind the formula above?

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Market clearing

- We have markets for $N + 1$ different goods types $n \in \{0, \ldots, N\}$
- We have I agents, each of whom would like to consume c_n^i
- To ensure markets are in equilibrium, what do we need to impose?
- The corresponding market clearing conditions are

$$\forall n \in \{0, 1, \ldots, N\} : \sum_{i \in I} c_n^i = \sum_{i \in I} y_n^i = \mathbf{y}_n$$

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Equilibrium prices

- Solution: letting $Q_n \equiv p_n / p_0$ and defining the aggregate endowment of good n as $\mathbf{y}_n \equiv \sum_{i \in I} y_n^i$ we can show

$$\forall n > 0 : Q_n = \frac{\gamma_n \mathbf{y}_0}{\gamma_0 \mathbf{y}_n}$$

  - The price we're dividing by (i.e. p_0) is called the numeraire
  - Why do we need to divide by p_0 instead of simply solving for it?

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Link to dynamic stochastic (general equilibrium) models

- Why should we care about static deterministic models?
  - Famous insight of Arrow (1964) and Debreu (1959): uncertainty and time can easily be incorporated in the previous framework!
- Specifically, we can re-interpret our model as one with
  - Two periods (for simplicity; no actual constraint on [illegible]
  - N possible future outcomes / states next period [illegible]
  - One type of consumption good (again, only for [illegible]

$$c_0^i = \text{consumption of agent } i \text{ in the in[illegible]}$$

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- We can also set the utility weights γ_n equal to (why?)

$$\gamma_0 = 1 \qquad \gamma_n = \beta\pi_n$$

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A DSGE model

- Agent i's optimisation problem then becomes

$$\max_{c_0^i, c_1^i, \ldots, c_n^i} \left\{ \log\left(c_0^i\right) + \beta \sum_{n \in \{1,\ldots,N\}} \pi_n \log\left(c_n^i\right) \right\}$$

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- What is the sum in the optimised expression equal to?

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Arrow securities

- Defining $a_n^i \equiv c_n^i - y_n^i$ the constraint can be rewritten as

$$c_0^i + \sum_{n \in \{1,\ldots,N\}} Q_n a_n^i = y_0^i$$

  - Since y_n^i are fixed, choosing c_n^i is equivalent to choosing a_n^i
- Can think of the agent as choosing c_0^i and holdings of assets a_n^i paying a unit of consumption only in state n (next period)
  - These assets are known as Arrow securities and their prices are denoted by Q_n. What is the unit of account?

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Asset market completeness

- Asset market completeness implies no difference between
  - a dynamic stochastic model
  - a static model in which consumption at all possible future dates / states is chosen in the initial period
- But what does it imply about the number of Arrow securities?
  - Would you consider this to be a strong assumption?
- In summary, and as noted by Townsend (1979) (and many others), the insights of Arrow (1964) and Debreu (1959) are double-edged!
  - It seems there are few contingent dealings among agents relative to those suggested by the theory!
- We will stick to the complete markets assumption
  - Financial frictions consitute a popular deviation
  - Covered in more detail later in the course

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Asset market completeness

- Asset market completeness implies no difference between
  - a dynamic stochastic model
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- But what does it imply about the number of Arrow securities?
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Solving the dynamic stochastic problem

- How can we quickly solve the two-period DSGE model?
- Optimal consumption levels are given by

$$c_0^i = \frac{1}{1+\beta}\left(y_0^i + \sum_{m\in\{1,\ldots,N\}} Q_m y_m^i\right)$$

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with Arrow security prices equal to

$$\forall n > 0 : Q_n = \beta\pi_n \frac{\mathbf{y}_0}{\mathbf{y}_n}$$

- To back out equilibrium security prices Q_n we need the aggregate endowments $\mathbf{y}_i$, discount factor β and state probabilities π_i

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Heterogeneous vs representative agent models

- We could also consider the following representative agent model

$$\max_{c_0, a_1, \ldots, a_n} \left\{ \log\left(\mathbf{c}_0\right) + \beta \sum_{n>0} \pi_n \log\left(\mathbf{c}_n\right) \right\}$$

$$s.t. \quad : \quad \mathbf{c}_0 + \sum_{n \in \{1, \ldots, N\}} Q_n \mathbf{a}_n = \mathbf{y}_0$$

  - Note that i has vanished, we have one agent only!

- What is the solution for $\mathbf{c}_i$? What is the implication for $\mathbf{a}_i$?
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Notes on equivalence

- If all we're interested is aggregate prices then we can use the representative agent model...
  - We can think of there being a heterogenous agent economy in the background in which Arrow securities are actively traded
- Conditions under which the equivalence result holds were studied by Terence Gorman (Econometrica, 53)
  - Issue: the individual endowment distribution y_i should not matter for equilibrium prices
  - Idea: come up with conditions which guarantee that all agents, irrespective of wealth, chose the same bundle of goods
  - Necessary and sufficient condition: individual preferences admit Gorman-form indirect utility
- Assumption is satisfied by CRRA utility functions
  - log preferences are OK, but many other ones are not

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The consumption Euler equation

- We have previously shown that the price of the n'th Arrow security equals

$$\forall n \in \{1, \ldots, N\} : Q_n = \beta \pi_n \frac{\mathbf{y}_0}{\mathbf{y}_n}$$

- How could you use this formula to determine the price of an asset which pays a unit of consumption with certainty in the final period?
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- Letting E_0 be the expectation operator, we have

$$Q = \sum_{m \in \{1,\ldots,N\}} Q_m = \sum_{m \in \{1,\ldots,N\}} \beta \pi_m \frac{\mathbf{y}_0}{\mathbf{y}_m} = \beta \mathbf{y}_0 E_0 \frac{1}{\mathbf{y}_{t=1}}$$

- These derivations were for log utility where $u'(c) = 1/c$; using market clearing ($\mathbf{y} = \mathbf{c}$) the general expression for Q is

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The consumption Euler equation: intuition

- Define the net real interest rate r as

$$1 + r \equiv \frac{1}{Q}$$

- The consumption Euler equation can then be rewritten as

$$u'(\mathbf{c}_0) = \beta E_0 u'(\mathbf{c}_{t=1})(1 + r)$$

  - The 'utility' cost of a marginal increase in sav[illegible]
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- In models with inflation, the Fisher parity (an identity linking real and nominal interest rates and inflation)

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$$u'(\mathbf{c}_0) = \beta E_0 u'(\mathbf{c}_{t=1}) \frac{1 + i}{1 + \pi_{t=1}}$$

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- Importantly, increases in the nominal interest rate i would lead to lower consumption today, in line with the standard interest rate channel of monetary policy transmission
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- We started by solving a heterogenous-agent, static, deterministic, general equilibrium model
- We showed that when asset markets are complete, the setup can easily be made dynamic (i.e. account for many periods) and stochastic (i.e. account for uncertainty)
  - However, the assumption of complete markets seems [illegible]
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DSGE models: Summary (ctd)

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- Finally, we also derived the the Euler equation

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