AN EXERCISE ON CONDITIONAL FORECASTING

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The goal of these exercises is to compare two methods of computing the impact on GDP of a 25bp increase in government spending (every quarter, over three years). One of the methods will rely purely on a VAR, while the other will be based on a conditional forecast generated using the Kalman filter. The differences in answers generated by both methods reflect the different assumptions implicitly made about the underlying reduced form shocks and so illustrate the oft repeated platitude that 'shocks matter'. Please note that the model in the exercise is extremely simple and is *only* meant to illustrate the technique.

Exercise 1. A VAR(1) model for UK GDP growth and government spending. Use the data provided (VAR Data.xls) to estimate a VAR(1) model on UK GDP growth (subsequently denoted y_t) and government spending growth (g_t subsequently). Please make sure that the model includes a constant.

- What is the impact of government spending on GDP?
- What is the error term covariance matrix? Are the residuals uncorrelated?

Exercise 2. An unconditional forecast for UK GDP growth and government spending. Use the VAR to compute an unconditional forecast for GDP growth and government spending growth (for 10 years). We shall call this the base forecast, and we shall refer to the forecasts of y_t and g_t as the expected paths for GDP and government spending growth and denote them using $\widetilde{y}$ and $\widetilde{g}$ respectively.

Exercise 3. A VAR-implied conditional forecast for GDP growth. We are now interested in computing the impact on y_t of government spending growth g being 25bp higher than $\widetilde{g}$ for the first twelve quarters of the forecast period. Recalling that our VAR can be written as

$$\begin{bmatrix} y_t \\ g_t \end{bmatrix} = \begin{bmatrix} \mu_y \\ \mu_g \end{bmatrix} + \begin{bmatrix} f_{y1} & f_{y2} \\ f_{g1} & f_{g2} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ g_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^g \end{bmatrix}$$

we see that there are infinitely many combinations of the reduced form shocks ε_t^y and ε_t^g giving us the desired path for g. While the implied path of g will be identical for all of those paths of ε^y and ε^g, they may have very different implications for output y_t (as we will soon see).

- Assume that

$$\varepsilon_t^y \equiv 0$$

  and back out ε_t^g to get the desired path for g (equal to $\tilde{g}$ plus 25bp during the first three years).
- **NOTE:** in Eviews it turns out easiest to compute the conditional forecast for y first and then use that, along with the conditioning path for g to back out the implied ε_t^g.
- Use the shocks ε_t^g which you have just computed along with ε_t^y to compute the VAR-implied forecast for $\widehat{y}_t$ over the 10 year horizon.
- What do you need to do with $\widehat{y}_t$ to infer the impact of the increase in government spending growth?
- Comment on the size of the impact. Can you explain what is causing the change in y to be of this magnitude?

Exercise 4. A Kalman-filter implied conditional forecast for GDP growth. We shall now redo the exercise but instead of making assumptions about ε_t^g and ε_t^y directly, we shall use the Kalman filter.

- Use the Kalman filter to generate the forecast for $\widehat{y}_t$, *conditional* on government spending growth equal to $\tilde{g}$ plus 25bp during the first three years.
- Now compute $\widehat{y}_t - \widetilde{y}_t$ and compare the estimate of the impact of higher government spending on GDP growth to that computed in Exercise 3 (ensure you are recovering the conditioning path for g). Are they similar? What can account for the differences?

Exercise 5. Shocks matter!

- Back out the paths of ε^y and ε^g implied by the Kalman filter conditional forecast and compare them to those used in the VAR exercise.
- Which of the two shock profiles seems most different from that used in the VAR exercise?
- Is that the one which is leading to the differences in the conditional forecasts for $\widehat{y}_t$ between the two methods (this can be easily inferred without doing any calculations)?
- Do you understand what is going on? Discuss.