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ABSTRACT

This study examined the determinants of the REER and the effect of monetary policy on the REER. The study utilized fractional cointegration VAR (FCVAR) and Bayesian VAR models based on data covering the period 2006 to 2021. The results suggest that the Zambian REER is fractionally cointegrated with the real copper price, fiscal deficit and productivity differential. The study also shows that a shock to the equilibrium exchange rate tends to have a slow speed of adjustment to its steady state. The results from the Bayesian VAR show that a positive monetary impulse leads to the REER appreciating and gets to the equilibrium level after 15 quarters. The major policy implication is that there is some scope for policy intervention to hasten the return to equilibrium when the economy is buffeted by shocks that result in non-competitive prices. In addition, monetary policy tightening leads to a rise in the productivity differential against Zambia (as domestic productivity declines) implying that monetary authorities should make a trade-off between high inflation and loss of productivity for about a two year-period.

I. Introduction

The Real effective exchange rate (REER) is an important macroeconomic variable, playing a pivotal role as an indicator of international competitiveness of a nation in comparison with its trade partners. As a gauge of competitiveness, the REER not only influences but also determines the performance of the export sector Click or tap here to enter text.Given the allocative and competitive functions of the REER, institutions like the International Monetary Fund (IMF) typically advocate for developing and emerging economies to maintain the actual REER in close proximity to the equilibrium real exchange rate (EREER).[1]

In the 1990s, Zambia undertook economic reforms, leading to the liberalization of the current and capital accounts. Consequently, the exchange rate emerged as a pivotal factor influencing short-term capital flows, the dynamics of exports and imports of goods and services, and the maintenance of external equilibrium. Additionally, fluctuations in the exchange rate exert an impact on inflation through pass-through effect on imported goods. Therefore, a deep understanding of the factors determining the equilibrium real effective exchange rate is essential for effective economic management in general and particularly in the formulation of exchange rate policies. Previous studies on the REER on Zambia have assumed integer order of integration of the data. Therefore, none of these studies have explored the role of long-range dependency in the data and thereby establish whether the Zambian REER and its determinants are fractionally integrated. This is in view of recent empirical evidence showing that most economic and financial variables have long memory (Tarasova and Tarasov, 2018 and Baillie, 1996).

It is against this background that this study sought to examine the evolution of the nominal exchange rate, determinants of the REER and impact of monetary policy on the REER.

Understanding long memory processes is crucial because these processes display nonintuitive properties where many familiar mathematical results fail to hold (Graves *et al.*, 2017). Failure to recognize fractional cointegration in this regard may result into generating policy relevant information (forecasts and or structural analytics) that could lead to policy regrets.

While most of the studies on Zambia have not focused on a specific period of notable economic development, this study focuses on the period between 2006 and 2021 known as the post Highly Indebted Poor Country Initiative (HIPC) debt relief era. The period from 2006 to 2012 was characterized by high economic growth while subsequent periods were identified with low growth as well as unsustainable fiscal deficit and debt levels thereafter with possible implication on the REER. Additionally, studies on the REER in most Sub-Saharan countries have utilized methods based on integer order of integration. This study adds value by using the fractional cointegration, which has not been previously used in Zambia.

The rest of the study is structured as follows. Section 2 provides evolution of the exchange rate while section 3 outlines theories of determination of the equilibrium real effective exchange rate as well as the empirical literature review. Section 4 describes the methodology and data sources. The results and discussion are presented in section 5 and section 6 concludes the study.

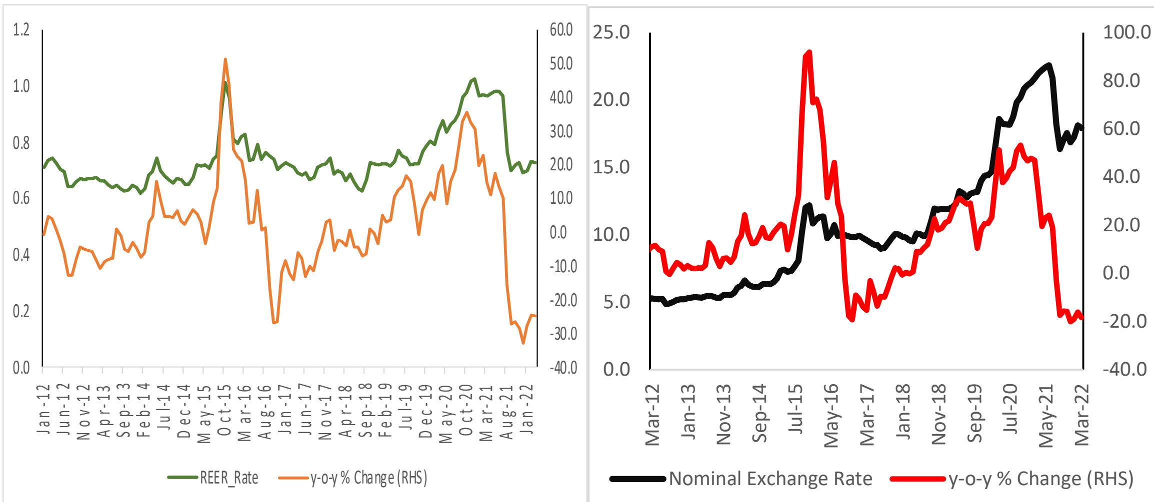
II. Recent Developments in the Nominal and Real Effective Exchange Rates

Over the period January 2012 – March 2022, the REER has undergone two major depreciations followed by sharp corrections (Figure 1). The first period, June – October 2015, the depreciation of the REER was largely occasioned by the depreciation in the nominal exchange rate following monetary policy normalization and ending the unconventional monetary policy (UMP) – previously pursued by US Federal Reserve Bank (FRB) in response to the 2008 global financial crisis. The eventual rise in interest rates by

[1] The EREER is defined as the value of the REER that aligns with a simultaneous realization of internal and external equilibrium.

the FRB and cessation of its asset purchase programme, resulted in a decrease in commodity prices and capital outflows from frontier and emerging market economies. Commodity backed currencies like the Kwacha were hit hard leading to marked depreciations during the period as terms of trade deteriorated.

Figure 1: REER and Nominal Exchange Rate Developments



The depreciation of the Kwacha also led to the rise in inflation. The second depreciation episode lasted from May 2020 to November 2021. This was largely due to the nominal exchange rate depreciation owing to rising debt service amidst declining reserves and reduced foreign exchange supply. However, in the month of December 2021 there was a sharp appreciation in the REER which can be traced to the appreciation in the nominal exchange rate that benefited from improved foreign exchange supply owing to high copper receipts in view of rising commodity prices at the time as well as higher inflows from non-resident investors in Government securities and positive market sentiments that arose from better macroeconomic prospects, including expectations and eventual allocation of the Special Drawing Rights (SDRs) by the IMF. The appreciation of the exchange rate in 2016 was also due to portfolio debt inflows from non-resident investors in Government securities, and constrained aggregate demand due to the tight monetary policy stance, Bank of Zambia Annual Report (2016)..

III. Theories of Equilibrium Real Effective Exchange Rate Determination and Empirical Review

In this section, we discuss various theories of equilibrium REER determination, their derivation, key variables used and the associated strengths and weaknesses.

Purchasing Power Parity

Since the pioneering work of Cassel (1918) , the Purchasing Power Parity (PPP) concept has been widely discussed in several research works. Its basic argument hinges on a proposition known as the law of one price (LOP), which provides a building block to variations of PPP. The LOP argues that without impediments to international trade and capital flows (such as tariff barriers and capital controls); with both domestic and foreign economies operating at full employment level in a market-based price system, identical goods (i) sold in different countries must sell for the same price if expressed in the same currency.

$$P^i = P^{*i}.E \tag{1}$$

where $\boldsymbol{P^{i}}$ is domestic price, $\boldsymbol{P^{*i}}$ is foreign price, and $\boldsymbol{E}$ is the PPP exchange rate.
The concept of PPP generalizes the LOP proposition and asserts several important claims. First is that general price level of a basket of similar products in different countries will always be equalized when they are measured in a common currency (a concept known as the absolute PPP) as shown in Equation (2) below:

$$P = P^{i}.E \tag{2}$$

Put differently, the central notion of PPP theory claims that price levels determine equilibrium exchange rate (Equation 3). Ceteris Paribas, a rise in local price level should raise $\boldsymbol{E^{PPP}}$, i.e., a depreciation of the local currency. All things being equal, a rise in the domestic price level relative to the foreign price level is expected to raise $\boldsymbol{E^{PPP}}$ **,** which implies a depreciation in the domestic currency.

$$E^{PPP} = \left(\frac{P}{P^{*}}\right) \tag{3}$$

Secondly, the PPP real exchange rate is always constant and equals to one. This can easily be shown by substituting Equation (3) into the equation of the real exchange rate, $Q = \left(\frac{\boldsymbol{E.P^{*}}}{\boldsymbol{P}}\right)$ and solve for the real exchange rate (Q). Given the basic assumption of price rigidity in the short run, the prevailing nominal exchange rate may however deviate from the PPP rate. Thus, the PPP is often referred to as the long-run equilibrium exchange rate. When $\left(\frac{E}{E^{PPP}}\right) > 1$, the local currency is undervalued, and if $\left(\frac{E}{E^{PPP}}\right) < 1$, an overvaluation has taken place. The presence of stickiness in nominal prices, coined in the seminal work by Dornbusch (1976) on the overshooting exchange rate, led to popular arguments that temporary deviations from the PPP should take place in the short-run, but one would, at least theoretically, expect convergence in the long-run. Frankel (2007) argues that one way to deal with a sufficiently slow convergence to purchasing power parity rate is to have observations over a long span of period.

Due to the simplifying assumptions regarding the PPP, there are a number of other limitations in explaining the shocks or changes that impact the nominal and real equilibrium exchange rate. In addition, the LOP may not hold due to a number of factors such as trade inhibitions (existence of tariff and non-tariff barriers), capital controls, and administered pricing.

Even when LOP holds, changes in shares or weights of different sectors or products may lead to the failure of PPP. In the long run, due to a series of factors, consumers may acquire different preferences and producers choose different products to maximize profits. The weights thus vary overtime in both domestic and foreign markets, and they may diverge significantly. As a result, PPP in this case is a flawed measurement as the baskets of goods being compared may consist of completely different products, in turn, violating the assumption of identical basket of goods on which the theory is based. The existence of non-tradable goods also undermines the validity of the PPP concept. Hence, PPP fails to capture shocks in the non-tradable sector resulting in persistence divergences between PPP and the prevailing real exchange rates.

The Fundamental Equilibrium Exchange Rate (FEER)

The concept of the fundamental equilibrium exchange rate (FEER) attempts to capture other factors beyond monetary variables in determining equilibrium exchange rates (Williamson, 1994). This phenomenon is built on the real effective exchange rate simultaneously securing internal and external balances[2]. Several factors determine the level of the FEER among them potential output growth, potential gaps in productivity growth and sustainable current account (Balassa, 1964).

[2] Internal balance is attained when the economy is at full employment output and operating in a low inflation environment while external balance is characterized as a sustainable balance of payment position over a medium-term horizon, ensuring desired net flows of resources and external debt sustainability.

Generating a FEER involves two sequential steps, that is, identifying the external balance by equating the current account balance (CA) to capital account balance (KA) as specified in equation 4 (Clark and MacDonald, 1998; MacDonald, 2000)..

$$CA = -KA \tag{4}$$

The current account, using the FEER approach can be expressed as follows:

$$CA = ntb + nfa \tag{5a}$$
$$ntb = \delta_0 + \delta_1 q + \delta_2 y_d^- + \delta_3 y_f^- \tag{5b}$$

Where,

$\delta_1 > 0$, $\delta_3 > 0$, and $\delta_2 < 0$

$$nfar = f(q) \tag{5c}$$

where CA is the current account, $\boldsymbol{nfar}$ is returns of net foreign assets, $\boldsymbol{ntb}$ **is** the net trade balance, $\boldsymbol{y_d^-}$ and $\boldsymbol{y_f^-}$ are the full employment outputs of both the local and foreign economies, and q is the real effective exchange rate.

In most applications of the FEER approach, the level of equilibrium capital account $\boldsymbol{KA}^*$ over the medium term is exogenously determined. It is important to underline here that $\boldsymbol{KA}^*$ excludes speculative capital flows. The medium-term balance of payment equation (6) is generated by combining equations 4, as well as 5a to 5c holding the basic assumptions above:

$$CA = f\left(q^{REER}, y_d^-, y_f^-\right) = -KA^* \tag{6}$$

There are three vital elasticities that underlie the FEER approach, namely: the elasticity of the current account (CA) to domestic activity $(\boldsymbol{y_d^-})$, elasticity of CA to foreign output **(** $\boldsymbol{y_f^-}$**),** and the elasticity of CA to the real (effective) exchange rate $(\boldsymbol{q^{REER}})$. Given an assumption of full employment, outputs of the local and foreign economies, $(\boldsymbol{y_d^-})$ and $(\boldsymbol{y_f^-})$ respectively, and the medium-term equilibrium $(\boldsymbol{KA}^*)$**,** the level of real exchange rate, FEER, is derived from equation (6). The $\boldsymbol{q^{REER}}$ obtained from equation 6 ensures a sustainable current account or a path to achieve macroeconomic balance—internal and external.

$$q^{REER} = f\left(KA^*;\ y_d^-;\ y_f^-\right) \tag{7}$$

The key criticism of the FEER approach is that the size of currency misalignment estimated by FEER is likely to be inaccurate largely on account of the assumption of sustainable current account or external balance that are considered unrealistic when used to calculate trade elasticity, a pitfall pointed out by MacDonald (2000). In addition, relying too much on the trade elasticity may generate an inaccurate estimate of the FEER trajectory. Further, a depreciation of real exchange rate in the domestic currency would not only lead to an improvement in the net trade balance but may also increase the return on net foreign assets. If the FEER only captures the changes in net trade balance while assuming an exogenously determined impact on the return on net foreign assets, then the size of FEER appreciation may be overestimated.

The Behavioral Equilibrium Exchange Rate (BEER)

The BEER approach is an improvement of the FEER approach in that it incorporates the cyclical and temporary changes as determinants of the exchange rate. Hence, the underlying assumption of macroeconomic balance of the FEER approach is absent under the BEER approach. The theoretical foundations of the BEER approach are underpinned by the works of MacDonald (1997) and Clark and MacDonald (1998), which allow for the short-run or temporary analysis of the real exchange rate, and rests on the concept of uncovered interest rate parity (UIP):

$$E_t(e_{t+1}) - e_t = i_t - i_t^* \qquad (8)$$

Where $E_t(e_{t+1})$ represents the expected value of the nominal exchange rate in period, **t,** for period **t+1**. $\boldsymbol{e_t}$ is the nominal exchange rate at period t, defined in terms of domestic currency per unit of foreign currency. Thus, a rise in $\boldsymbol{e}$ implies a depreciation of the local currency. $\boldsymbol{i_t}$ and $\boldsymbol{i_t^*}$ denote local and foreign nominal interest rates, respectively. The risk premium component is assumed to be absent in Equation (8), as the primary objective here is to provide a simple construction of the BEER approach to equilibrium exchange rate determination.

Subtracting the expected inflation differential from both sides of equation (8), yields the real interest parity as shown in equation (9) [3]

$$E_t(q_{t+1}) - q_t = r_t - r_t^* \qquad (9)$$

Where,

$\boldsymbol{E_t(q_{t+1})}$ denotes the expected real exchange rate for period t+1
$\boldsymbol{q_t}$ is the observed real exchange rate, r_t is the domestic real interest rate[4], r_t^* =is the foreign real interest rate[5].

Rearranging equation (9), the observed real exchange rate $\boldsymbol{q_t}$ can be represented as a function of the expected value of real exchange rate $\boldsymbol{E_t(q_{t+1})}$, and the current real interest rate differential.

$$q_t = E_t(q_{t+1}) - (r_t - r_t^*) \text{ -} \qquad (10)$$

Under the BEER approach, the unobservable expectation of real exchange rate, $\boldsymbol{E_t(q_{t+1})}$, is assumed to be determined solely by long-run economic fundamentals **(Zt)** . In other words, the BEER approach produces estimates of equilibrium real exchange rate $\boldsymbol{q_t^{BEER}}$ which incorporates both the long-run economic fundamentals $E_t(q_{t+1}) = f(Z_t)$ and the short run interest rate differentials such that

$$q_t^{REER} = f(Z_t, (r_t - r_t^*)) \qquad (11)$$

Different authors have used various variables to represent the vector **(Zt).** For instance, Clark and MacDonald (1998) assumed three long-run determinant variables of vector **(Zt)**, are terms of trade **(tot)**, the relative price of non-traded to traded goods **(tnt)** , and net foreign asset **(nfa).** Thus,

$$E_t(q_{t+1}) = f(tot_t^-, tnt_t^-, nfa_t^-) \qquad (12)$$

[3] $(E_t(p_{t+1}) - p_t) - E_t(p_{t+1}^*) - p_t^*) = E_t(\Delta p_{t+1}) - E_t(\Delta p_{t+1}^*)$
[4] $r_t = i_t - E_t(\Delta p_{t+1})$

[5] $r_t^* = i_t^* - E_t(\Delta p_{t+1}^*)$

Clark and MacDonald (1998) added the role of the time-varying risk premium component to the uncovered interest parity [6] . Combining equation (11), equation (12) and the risk premium component, the BEER is expressed as:

$$BEER = f\left((r_t - r_t^*), tot, tnt, nfa, \left(\frac{gdebt_t}{gdebt_t^*}\right)\right) \tag{13}$$

Review of Empirical Literature

Hau (2002) empirically employed a cross-sectional analysis on 48 (25 non-OECD and 23 OECD) countries. The author used the real effective exchange rate (REER) volatility measured as the moving sample standard deviation of REER percentage changes over a three-year-period. The study found that economic openness and real exchange rate volatility were inversely related. Another estimation was implemented for OECD countries only, based on the premise that they were more homogenous.[7] The results also showed a negative relationship between real exchange rate volatility and trade openness.

Mathisen (2003) demonstrated that for Malawi different components of public expenditure had varying effects on the real effect exchange rate. The study showed that an increase in government spending, excluding salaries and wages led to a real appreciation. On the other hand, an increase in government spending on salaries and wages exerted pressure on the current account, and occasioned a real depreciation. The study also found that real per capita growth also exerted an influence on the real effective exchange rate.

It is nonetheless worth noting in view of the findings on Malawi about the effect of fiscal operations (based on different components) on the REER that mixed results have been established for developing countries on the relationship between fiscal deficit and the REER. In a panel estimation involving developed and developing nations, Nguyen et al., (2020) found a negative relatrionship between government spending (military spending to be specific) and the REER for developing countries while the relationship established for developed countries was a positive one. The negative relationship has some theoretical support either from the real business cycle model or the Keynessian counterpart as documented by Moreno Badia and Segura-Ubiergo (2014).

Mungule (2004) found that terms of trade, closeness of the economy and the nominal devaluation are important factors that determine the level of the real effective exchange in Zambia. While excess supply of domestic credit and the growth in nominal exchange rate/ nominal devaluation were important short-run determinants. Additionally, a univariate analysis of Zambia's REER by Phiri (2020) established that the Zambian REER is non-linear and mean reverting.

Vural (2018), using the VECM framework found that the productivity differentials largely determined the Turkey equilibrium real effective exchange rate. An increase in turkey's productivity relative to trading partner countries occasioned an appreciation of the REER. However, the study found that an increase in real oil prices led to an appreciation, contrary to an expectation of a real depreciation.

Eckstein and Friedman (2011) found that relative per capita GDP and terms of trade were the most significant factors that explained Israel's real exchange rate fluctuations and trends. Other variables such as government expenditure and the dependency ratio did not impact the real effective exchange rate.

[6] The ratio of domestic and foreign government debt $\left(\frac{gdebt_t}{gdebt_t^*}\right)$.

[7] This was based on the notion that these countries experience similar shocks that are relatively similar in magnitude.

Studies on the REER using fractional integration techniques have become common due to the possibility that economic and financial variables being characterized by the phenomenon of long memory.[8] The case of long memory is one where past observations in a given variable are non-trivially correlated with distant future counterparts (Campbell et al., 2007). This implies that such variables may be fractionally integrated as opposed to the commonly held view that they have an integer order of integration. The time series realizations with long memory are characterized by autocorrelation function which exhibits persistence that is neither consistent with an I(1) process nor I(0) process (Baillie, 1996). In other words, variables with long memory and thus fractionally integrated are characterized by slowly decaying autocorrelation functions (Campbell et al., 2007).

An early study to explore the possibility of fractional integration and cointegration among real exchange rates (RER) established the presence of fractional integration among developed countries and confirmed their fractional cointegration, (Cheung and Lai, 1993). Although this appears to be the case for the industrialized countries, it does not preclude a test for the same for developing countries, since for most of these countries, their real effective exchange rates are linked to foreign inflation as well as nominal exchange rates by virtue of trade. Therefore, there could be a possibility that developing countries REER variables may be fractionally integrated. This is through their linkage with developed countries' variables since interaction between economic variables are influenced by the memory function that connects them (Tarasov and Tarasova, 2018).

One of the other early studies to consider fractional integration in the real exchange rate and make conclusions about the fractional cointegration of the variables was done by Holmes, (2002). The study sought to establish whether the PPP based on the Real Exchange Rate (RER) for developing countries that included some sub-Saharan Africa countries (SSA) hold. The Geweke and Porter-Hudak (GPH)'s semi-parametric method was used. It was established that the PPP holds for eight countries including South Africa from the SSA countries in view of the respective real exchange rates being fractionally integrated. The author concluded that the spot exchange rate, foreign prices, were fractionally cointegrated. However, it is our considered view that Holmes (2002) view may be too strong a conclusion on fractional cointegration in the absence of formal test for fractional cointegration as for instance suggested by Johansen and Niels (2012) and implemented by Gunay (2018) and Salisu *et al.* (2020) on precious metal prices and Islamic stocks, respectively.

Caporale and Gil-Alana (2004) also tested for fractional cointegration of the real exchange rate with other variables – real and monetary variables. They argue in the paper that a fractionally integrated and cointegrated structure could be the best modelling approach for real exchange rates. They are of the view that the error correction term's response to shocks is slower than inferred by classical methods such as the Johansen (1995). They argue that the equilibrium errors obtained with classical methods may exhibit fractionally integrated property of displaying slow mean reversion. This is the case where the error correction term may show signs of a long memory process with deviations from equilibrium being highly persistent. This is the case, in our view, akin to relationships characterized by very slow speed of adjustment to the equilibrium when estimated with classical cointegration methods and may thus require the use of fractional cointegration techniques. The results by Caporale and Gil-Alana (2004) found the real exchange rates of the Deutsch Mark–US dollar and the Japanese Yen–US dollar pairs to be fractionally cointegrated with real interest rate and labor productivity differentials.

[8] See, for example, Baillie (1996), Škare and Stjepanović (2013), Tarasov and Tarasova (2016), Ngene, Tah and Darrat (2017), Tarasova and Tarasov (2018), Zheng, Liu and Li (2018), Tarasov and Tarasova (2018), Mensi, Tiwari and Al-Yahyaee (2019) and Nguyen et al., (2020).

Similarly, Gil-Alana and Trani (2019) employed fractional integration technique to examine whether the G7 trade weighted REERs were mean reverting, arguing, in support of the use of fractional integration methods, that changes in the REER are very persistent and this makes it difficult to detect stationarity, and therefore mean reversion, by using conventional univariate unit-root test techniques.

Regarding the interaction of monetary policy impulses with the REER, Ndung'u (1999) assesed whether the exchange rate of Kenyan currency is affected by monetary policy adjustments and if these effects were permanent or transitory. The real exchange rate was decomposed into cyclical and permanent components. Causality tests were performed between several measures of monetary shocks and the cyclical component of the exchange rate. The results showed that excess money supply affected the cyclical movement of the real exchange rate in the short run only, an indication that its effect is temporal. This is consistent with an earlier theoretical proposition and application in a study by Calvo, Reinhart and Végh (1995) on some Latin American countries. It has also been established and argued that the steady state real exchange rate is independent of permanent changes in monetary policy. They asserted that this result depends on the fact that there is no direct steady state link in theory between inflation and real exchange rate, implying that monetary effects are transitory.

In a recent study, and specifically on a SSA country, Iddrisu and Alagidede (2020) found monetary policy impulses to affect South Africa's in which case monetary policy tightening was accompanied by an appreciation in the REER. Similar results were found by Mundra and Bicchal (2023) but on India..

IV. Data and Methodology

Economic variables normally interact as a system where a shock on one variable affect others either contemporaneously or with a lag. Most of the studies done in sub-Saharan Africa especially on the equilibrium real effective exchange rate and monetary policy have mainly used classical approaches on time series where it is assumed that the time series properties follow the integer order of integration. However, as highlighted elsewhere in the study, most economic, and financial time series data often exhibit long memory, a feature that has significant implication for monetary implementation. This study therefore used fractional cointegration approach. The issue of fractional integration and fractional cointegration presents a host of models/methods available for testing this phenomenon. A detailed review of such methods has been well documented by Baillie (1996) with the accompanied application of the methods to analyze economic and financial data. The study also used Bayesian VAR with sign restrictions procedure to establish the dynamic impact of monetary policy shocks on the REER.

Estimation

Since our interest is to determine the long-run and short-run determinants of the REER for Zambia under the assumption that variables in the system may be fractionally integrated, as argued in modern literature on time series economic and financial data, a fractionally cointegrated VAR (FCVAR) model is used. This is a non-restrictive model when compared to the cointegrated VAR (CVAR) counterpart where restrictions of I(0) and I(1) are imposed in the assumption of the order of integration of the data. In this regard, for a given time series Y_t with a dimension of p, its FCVAR model of the error correction form (Johansen and Niels, 2012) is expressed as

$$\Delta^d Y_t = \alpha\beta'\Delta^{d-b}L_bY_t + \sum_{i=1}^{k}\Gamma_i\,\Delta^d L_b^i Y_t + \varepsilon_t \qquad (14)$$

where, ε_t is p-dimensional i.i.d. $(0, \Omega)$, Δ^d is the fractional difference operator, and $L_b = 1 - \Delta^b$ is the fractional lag operator. Long run parameters are represented by α and β, which in both cases are $p \times r$ matrices such that $0 \leq r \leq p$. In this set up, r is the fractional cointegration rank while β represents, by its columns, the r fractional cointegration vectors and α is the speed of adjustment to the equilibrium for each of the variables. The combination $\beta' Y_t$ represents the fractional cointegration of the variables in the system. The parameters d and b are the fractional parameters with d representing the fractional order of integration for the given time series while b determines the degree of fractional cointegration. In the model above, when $d = b = 1$, you obtain the Johansen (1995) CVAR model.

The estimates obtained by fitting the above model with Zambian data were used to assess the dynamic behaviour of the equilibrium REER relationship with its determinants and the medium-term outlook. Estimations were done using the *FCVAR* package obtained from the Comprehensive *R* Archive Network. The steps involved in the estimation procedure are as follows:

i. Undertake a lag length selection based on the likelihod ratio (LR) and or Akaike information criterion (AIC) in order to get a well behaved VAR;
ii. Carry out a Cointegration test (Trace test) using maximum likelihood methods and determine the number of cointegrating relationships based on the LR;
iii. Estimate the model(s) using information obtained by steps (1) and (2) above with parameter space for d and b ranging between 0.01 and 2.0. Other initial values incorporated; and
iv. Establishing the suitability of the FCVAR model (Johansen and Niels, 2012) vs CVAR model (Johansen, 1995).

To estimate the impact of monetary shocks on the REER, an unrestricted VAR (equation 15) was estimated to recover necessary parameters. Its impulses were allowed to interact with appropriate sign restrictions using Bayesian techniques to trace dynamic impulse response functions. Sign restrictions are only amenable in a Bayesian framework and that structural shocks can be identified by determining whether the signs of the corresponding IRFs are consistent with economic theory (Cha and Bae, 2011). Bayesian techniques are ideal for short observations and data with long memory (Harjes and Ricci, 2010; Carlin and Jonas, 1985). Furthermore, Bayesian techniques also take care of nonlinearities in the data, which may be present in long memory processes, (Kablan and Kaabia, 2018).

$$y_t = \alpha + \sum_{i=1}^{L} \boldsymbol{B}_i y_{t-L} + \varepsilon_t \tag{15}$$

where,

$t = 1, - - - - -, T$;
$\alpha = 1 \times \rho$ unknown vector of intercepts;
$\boldsymbol{B}_i =$ unknown $\rho \times \rho$ matrix of coefficients, $i = 1, - - - - -, L$; and
$\varepsilon_t = \varepsilon_1, - - - - -, \varepsilon_T$, the independent and identically distributed error terms.

The error terms are normal, i.e. $N_\rho(0, \theta)$ with $\rho \times \rho$ unknown covariance matrix $\boldsymbol{\theta}$.

Economic meaning may be extracted from the model above based on the property that a one step ahead forecast errors of a reduced form VAR is linked to structural innovations (Cha and Bae, 2011; Danne, 2015) under the relationship;

$$\boldsymbol{A}\varepsilon_t = e_t \tag{16}$$

where, $\boldsymbol{A}$ is an $n \times n$ matrix of structural parameters with e_t being the structural shocks such that $e_t \sim iid(0,1)$. The structural parameters can be recovered from the reduced form VAR by invoking on the system the property expressed in equation (17)

$$\boldsymbol{A}\boldsymbol{\acute{A}} = \Sigma = E[\varepsilon_t \acute{\varepsilon}_t] \quad (17)$$

The $E[\varepsilon_t \acute{\varepsilon}_t]$ can then be obtained by estimating the VAR using the Ordinary Least Square (OLS) method. Recovering the structural shocks from the estimated $\widehat{\varepsilon_t}$, requires the identification of $\boldsymbol{A}$, (Cha and Bae, 2011; Danne, 2015). To ensure the results are consistent with economic theory, sign restrictions are imposed also. However, the identification procedure that also takes into account sign restrictions will not be feasible with the traditional Cholesky decomposition method as it may yield inconsistent results. Notwithstanding, the use of Bayesian techniques can help generate impulse responses amidst sign restrictions that are consistent with economic theory (this is only possible with the use of Bayesian techniques Granziera, Moon and Schorfheide, 2018; and Danne, 2015). The Bayesian techniques for sign restrictions compute cumulative impulse responses and then check if the range of the impulse responses generated are compatible with the sign restrictions imposed. Successful draws are done based on an uninformative prior, usually the uniform prior using Bayesian methods.

The Bayesian VAR with sign restriction based on the Uhlig (2005) algorithm was adopted for this study to deal with the problem of establishing the dynamic impact of monetary policy impulses on the Zambian REER. The Bayesian estimation is achieved through a Markov Chain Monte Carlo (MCMC) process with the credibility of the procedure (how well the model is specified) determined by the number of rejected draws (Danne, 2015). Zero or few rejections means the estimation procedure is reliable, but too many rejections implies the model estimated is not better than alternative models that fit the data and satisfies sign restrictions (Fry and Pagan, 2011 and Danne, 2015).[9]

The Uhlig (2005) penalty function method was used with a positive (+) sign restriction imposed on the monetary policy rate while a negative (-) sign restriction was imposed on the nominal exchange rate in line with economic theory. No sign restriction was imposed on productivity differential and the REER because we wanted the outcome to be data determined and thus draw insights on the impact of tight domestic monetary policy on the REER and productivity differential for Zambia.

Data Description

The choice of the sample period from the first quarter of 2006 to the fourth quarter of 2021 was based on the characteristics of economic developments during this period. The variables used in this research are given in Table 1.

Table 1: Variable Description

S/N	Variable	Definition	Data Source
1.	The Real Effective Exchange Rate	This is defined as the nominal effective rate between Zambia and its five major trading partners (China, South Africa, United States of America, United Kingdom and the Euro Area) adjusted for inflation differentials	The data was obtained from the Bank of Zambia's research database

[9] See details of steps involved in Bayesian VAR with sign restrictions estimation in Danne, (2015)

2.	The real productivity differential	This is the difference between the geometric mean[10] of the per capital GDP of Zambia's five major trading partners and the Zambia's per capital GDP.	The data for Zambia's per capita GDP was obtained from the Bank of Zambia's research database whilst the data on per capital GDP for the major trading partners, except, China were obtained from the Eurostat database. The data on GDP for China was from the National Bureau Statistics of China[11]. The population figures for China were obtained from database maintained by the Atlanta Federal Reserve Bank while the exchange rate was obtained from the IMF website
3.	Real Copper Prices	This is the value of one metric ton of copper in USD divided by the value for 2012Q1.	The nominal values were obtained from the Bank of Zambia
4.	Fiscal Deficit	This was obtained by dividing total expenditure by total revenue with the value above 1 indicating a fiscal deficit. Otherwise, it is a fiscal surplus.	The data was obtained from the Bank of Zambia's research database
5.	Monetary Policy Rate[12]	This is the key interest rate used by the Bank of Zambia to signal the monetary policy stance to the markets.	The data was obtained from the Bank of Zambia's research database
6	Inflation	Computed as a log of the consumer price index	Data was obtained from the Zambia Statistical Agency.

Source: Authors

Statistical investigations of the REER suggest that the series is neither an I(0) nor an I(1) process, but one that is fractionally integrated given the fractal signal classification estimates that suggest that the series is an *fGn* process in view of the estimated result of $\beta < 1$ (Table 2) and the slow decay of its ACFs (Figure 2).[13] The possible determinants of the REER for Zambia, the real copper prices and the productivity differential are also not integrated of the integer order with the former being an anti-persistent *fBm* process ($\beta < 2$) while the latter is a persistent *fBm* process given that $\beta > 2$ (Table 2). The fiscal deficit is also an anti-persistent *fBm* type of process. The ACFs for the four variables equally decays slowly (Figure 2).[14]

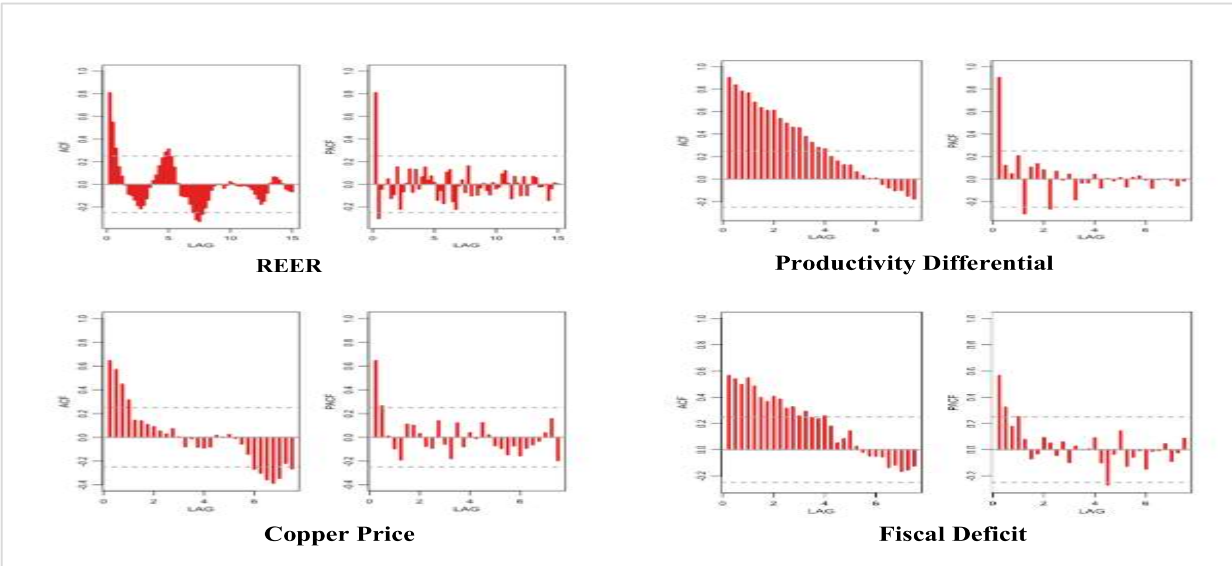
Table 2: Fractal Signal Classification

Variable	$\boldsymbol{\beta}$ Estimates	Fractal Signal Classification
Real Effective Exchange Rate (REER)	0.1367362	*fGn*
Real Copper Prices (Cup)	1.387995	Anti-persistent *fBm*
Productivity Differential (Prodif)	2.1234220	Persistent *fBm*
Fiscal Deficit	1.699726	Anti-persistent *fBm*

[10] The computation makes use of country trade weights used in the calculation of the real effective exchange rate.
[11] The GDP per capita for China was computed as the ratio of its GDP and population size while for the rest of the countries, GDP per capita was obtained as provided from the respective sources.
[12] Although the Monetary Policy Rate started in the second quarter of 2012, for prior periods, the 91-day treasury bill rate was used as a proxy for the stance of monetary policy.
[13] These are based on the R Package called PSD that uses Power Spectral Density (PSD) or periodogram approach . We specifically employ the *low*PSD option in the package. Details of the PSD estimation procedures are documented by Schaefer *et al*, (2014), Stadnitski (2012) and Eke *et al* (2002).

[14] See Annex 2 for details on the literature review on fractional cointegration

Figure 2: The ACFs and PACFs



V. Discussion of Empirical Results

Tests for Cointegration and Estimations

To get appropriate variables and an optimal model that explains the REER for Zambia in the context of its possible determinants a general to specific estimation strategy was employed. [15] The satisfactory combination was achieved with the model for five variables (four macroeconomic and one dummy) having a lag of one, although the AIC and LR favoured a model with three lags . The choice of one lag was settled for in the spirit of parsimony given a relatively short span of the data and small number of observations.

The trace test for parsimonious model also showed that two cointegrating equations exist (Table 2a). . However, when the estimations imposed a rank of one to just focus on the variable of interest being the REER as the dependent variable and the real copper prices, fiscal deficit, productivity differential and the dummy variable as independent variables resulted into a well-behaved estimated model.

Table 2a: Likelihood Ratio Tests for Cointegrating Rank (Lags = 1)

Dimension of system:		3	Number of observations in sample:		64
Number of lags:		1	Number of observations for estimation:		64
Restricted constant:		No	Initial values:		0
Unestricted constant:		No	Level parameter:		Yes
Rank	**d**	**b**	**Log-likelihood**	**LR statistic**	**P-value**
0	0.500	0.500	306.544	39.329	0.000
1	0.478	0.478	320.971	10.464	0.033
2	0.687	0.678	325.455	1.496	0.607
3	0.574	0.754	326.203	…...	…...

The results involving the five variables (REER, real copper prices, productivity differential, fiscal deficit and a dummy) shows that cointegration relationship of fractional order ($d = 0.167$) exists among the four

[15] This involved the terms of trade, productivity differential, interest rates differential, fiscal deficit, net foreign assets, and foreign reserves.

economic variables and they move together as a stationary process given that the fractional parameter d is less than 0.5 (Table 3a).

The characteristics roots are within the unit circle (Annex 3a). The two independent variables; copper prices and productivity differential have a negative relationship with the REER as expected. A rise in copper prices is more likely going to lead to an appreciation in the REER, which may be possible through the appreciation of the nominal exchange rate. A rise in copper prices could be a signal to foreign portfolio investors that Zambia dollar liquidity may be augmented through the increase in foreign exchange supply in form of mining tax payments and other domestic obligations. This provides an incentive for investors to increase their exposure to Kwacha denominated assets and thereby augmenting foreign exchange supply, leading to the appreciation of the nominal exchange rate, and subsequently the REER.

While empirical estimates show a negative coefficient for fiscal deficit, implying a rise in fiscal deficit is followed by an appreciation in the REER. This is not surprising as it is consistent with the mixed results established in empirical literature especially for non-developed countries data such as Zambia's. The implication is that Zambia's fiscal deficit has been to an extent financed by foreign borrowing. Between 2007 and 2021, Zambia's public external debt increased to US$13.04 billion from US$1.11 billion in 2006 following the HIPC completion point in 2005. Some of these debt flows were in form of portfolio flows that included the euro bond amounting to US$3.0 billion and foreign investors exposure to Government securities estimated to be US$2.96 billion at end December 2021.

Table 3a: Fractional Cointegration Results for REER, Productivity Differential, Real Copper Prices, and Fiscal Deficit

Dimension of system:	5	Number of observations in sample:	64
Number of lags:	1	Number of observations for estimation:	64
Restricted constant:	No	Initial values:	0
Unrestricted constant:	No	Level parameter:	Yes
Starting value for d:	0.800	Parameter space for d:	(0.010 , 2.000)
Starting value for b:	0.800	Parameter space for b:	(0.010 , 2.000)
Cointegrating rank:	1	AIC:	508.973
Log-likelihood:	-214.486	BIC:	595.328
log(det(Omega_hat)):	-7.487	Free parameters:	40

Fractional Parameters

Coefficient	Estimate	Standard error
d	0.167	0.061

Cointegrating equations (beta):

Variable	CI equation 1
Var1 (Real Effective Exchange Rate)	1.000
Var2 (Real Copper Prices)	0.004
Var3 (Productivity Differential)	0.008
Var4 (Fiscal Deficit)	0.002
Var54 (Dummy)	-0.369

Further, imposing a restriction $d = b = 1$, that is the three variables are best estimated by the CVAR model of Johansen (1995) as opposed to the FCVAR (the unrestricted model), the results show that the FCVAR is preferred to the CVAR (restricted) given a higher likelihood ratio of -214.486 for the unrestricted model compared to -240.470 in respect of the restricted counterpart (Table 3b). In addition, the restricted model has its characteristic roots falling outside the unit circle (Annex 3b). This is evident that the CVAR model may not be appropriate for the data involving the five variables used in this study.

Table 3b: Likelihood Ratio Test Results Between two Models

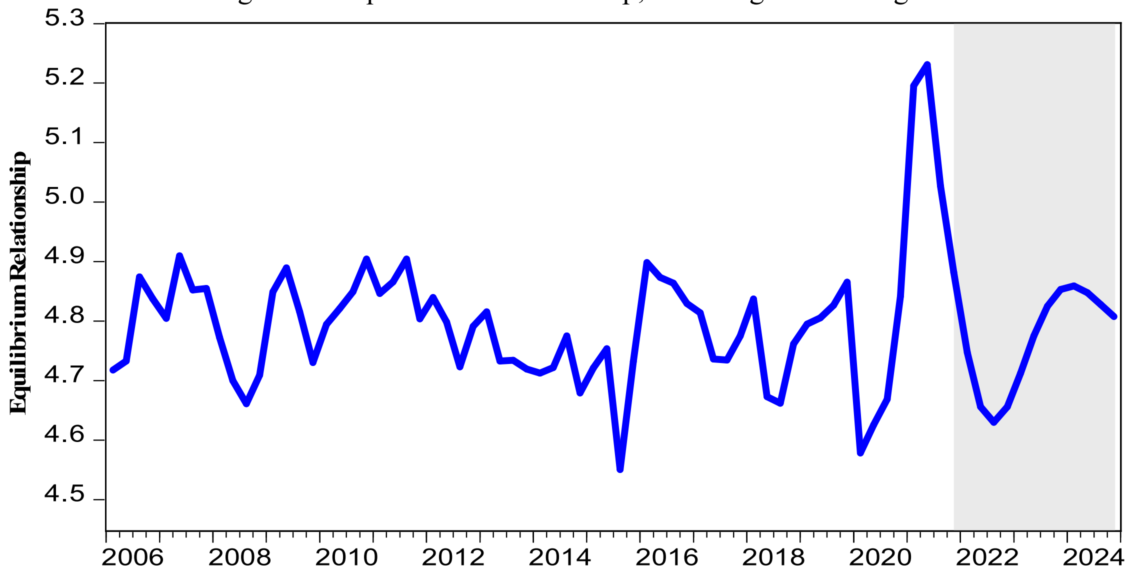
Unrestricted log-likelihood:	-214.486
Restricted log-likelihood:	-240.470
Test results (df = 1)	
LR statistic:	51.967
P-value:	0.000

The implication of these results is that the FCVAR model is appropriate in estimating the REER for Zambia during the period under review. This means that the real effective exchange rate for Zambia is fractionally cointegrated with its determinants as opposed to being of integer order of integration. Its long run equilibrium is primarily determined by the productivity differential between Zambia and its major trading partner countries, real copper prices as well as the fiscal deficit. The results also mean that a shock to the REER is neither transitory nor permanent. Equally, the effects of the shock to the productivity differential and or the real copper prices as well as the fiscal deficit does not have a permanent or a transitory effect on the REER over the sample period. This implies that shocks to the REER die out slowly as confirmed by the ACF in Figure 2. The results are somewhat in line with the findings by Phiri (2020) who established that Zambia's REER is non-linear.

The equilibrium relationship has an equally slow movement but stationary (Figure 4a, note that values are in logs). The stationarity in the equilibrium real effective exchange rate for Zambia can be seen if we give it some compact form with the bounds being in some epsilon neighbourhood of 4.54 (lower limit) and 5.24 (upper limit). This is even more apparent if we disregard the over shooting in the period 2021Q1 – 2021Q3 by letting the upper limit be in the infinitesimal neighbourhood of 4.95 as most of the observations will lie within the 4.54 – 4.95 corridor. The slow but stationary movement in the equilibrium REER is even more likely to continue three years ahead (2022Q1 – 2024Q4) as indicate in the forecasted trends in figure 4a. There is, therefore, a high likelihood that the equilibrium REER for Zambia could be indeed mean reverting but with a relatively slow speed of adjustment to its long run average.

To validate mean reversion behaviour of the equilibrium relationship and its slow speed of adjustment to its steady state value observed in figure 4a, we carried out a Beta (β) convergence estimation (See Annex 1 for the details including hypothesis testing). The choice of a Beta convergence procedure is to also get a sense of the long run steady state value around which the series making up the equilibrium oscillate. In this procedure, the negative sign of the β coefficient in the model used implies mean reversion in the series of interest. Further, the value of the β parameter ($\beta \in [-1, 0)$) which are possible values of a stationary or mean reversion process) indicates the speed of adjustment to the long run average (steady state value) while the ratio of the intercept (α) and the β coefficient (specifically, $-\alpha/\beta$) estimates the long run steady state value around which the equilibrium is established.

Figure 4a: Equilibrium Relationship, Including Forecasting.



The results confirm our suspicion. The equilibrium relationship is indeed a stationary process given the negative sign of the β coefficient and is characterised by a slow speed of adjustment to the long run average. The value of $\beta$is -0.37333 (i.e. $\beta = -0.37333$) while the steady state value is 4.80410361 (i.e. $-\frac{\alpha}{\beta} = 4.80410361$). Therefore, if there is a shock to a system of the three variables used in this study, the speed of adjustment of the equilibrium relationship to its long run average of 4.80 is 0.37 (or 37 percent), which is slow.

The result on the equilibrium REER's mean reversion behaviour is broadly consistent with the findings by Phiri (2020) on the actual Zambian REER which was established to be mean reverting in a univariate analysis. This was based on the exponential smooth transition autoregressive model whose estimated threshold values showed adjustment back to equilibrium.

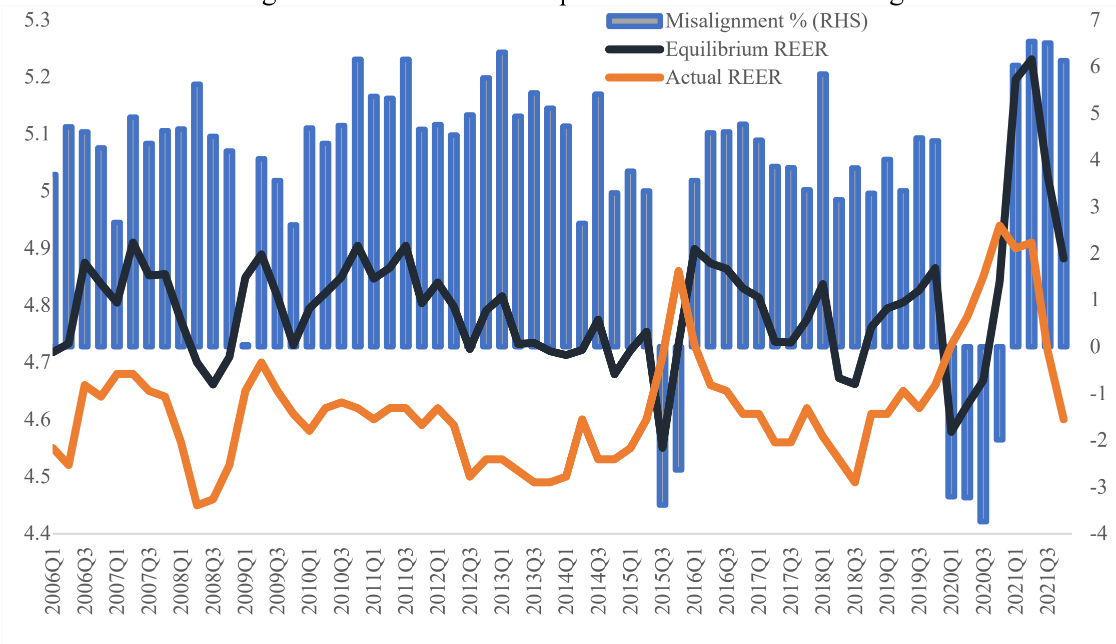
Misalignments of the REER

To get a sense of the extent of the misalignment of the actual REER from the estimated equilibrium REER presented above, the following metric was used to compute the values which are an indicator of the level of misalignment:

$$misalignment = \frac{\widehat{REER} - REER}{REER} * 100 \qquad (24)$$

The values obtained were compared to zero with an interpretation that observations above zero imply an over valuation of the REER whilst the values below zero can mean the case of undervaluation.

Figure 4b: Actual versus Equilibrium REER and Misalignment



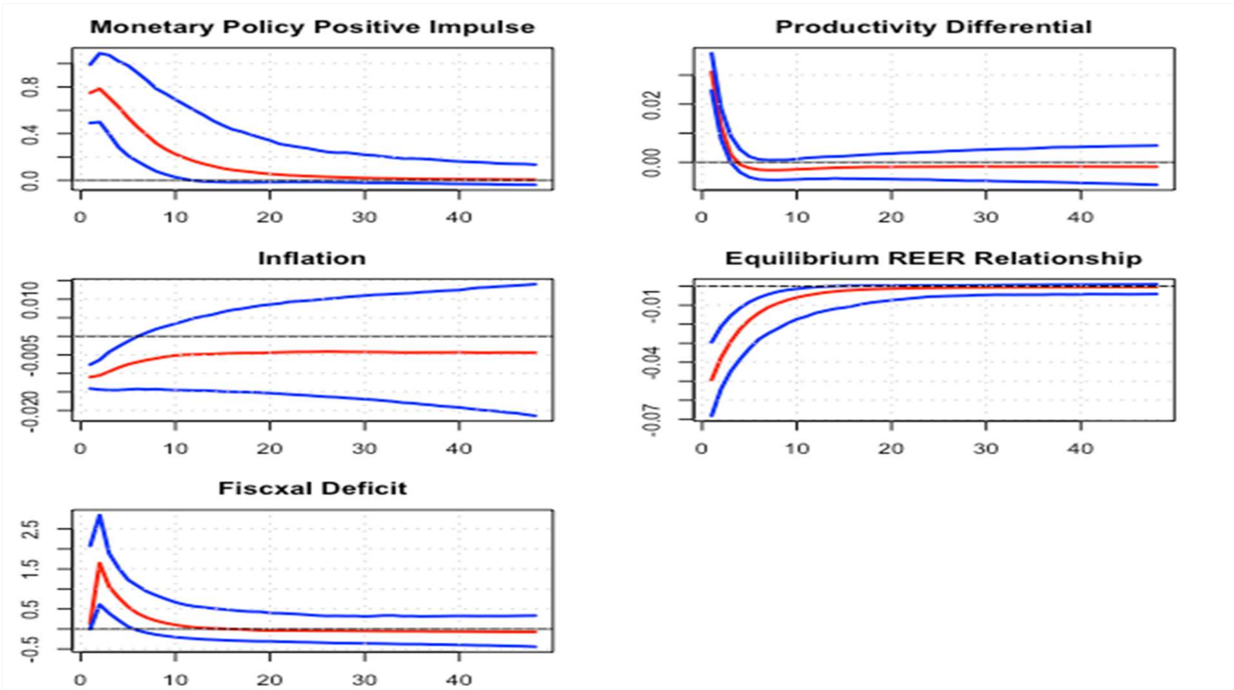
The results show that the REER has largely been overvalued in the post HIPC era with exceptions being only during the periods of globally induced shocks such as seen in the first quarter of 2009, third and fourth quarter of 2015 as well as the year 2020. The undervaluation cases identified coincides with the global financial crisis (GFC), the commodity price crisis that ensued when the Federal Reserve Bank normalised monetary policy with the end of the unconventional monetary policy (UMP) and the COVID-19 shock.

The GFC crisis caused some alignment in the Zambian REER mainly due to the depreciation of the nominal exchange rate by 21.6 percent in the first quarter of 2009, but this was followed by a 4.2 percent overvaluation in the second quarter of 2009. The overvaluation largely went on (with varying degrees) until the unwinding of the UMP which led to an undervaluation averaging 3.0 percent during the third and fourth quarter of 2020. The fall in commodity prices leading to the depreciation of the Kwacha largely explains this outcome. An undervaluation in the range of 2.0 to 3.2 percent was recorded during the COVID-19 period of 2020. The depreciation of the nominal exchange rate in the range of 3.1 to 20.6 percent over the four quarters primarily explains this development. The undervaluation was nonetheless followed by an overvaluation as the Kwacha due to an appreciation bias particularly in the last half of the year 2021.

Impact of Monetary Policy on the Equilibrium REER

To gain insights into how monetary policy is likely to impact the REER, a Bayesian technique employing sign restrictions was applied within a VAR Framework. This analysis incorporated the monetary policy rate, the REER and some of its determinants such as inflation, productivity differential and the fiscal deficit. The overall results show that a positive (i.e. contractionary) monetary policy shock leads to a decline in inflation, an appreciation of the REER, and an initial rise in fiscal deficit. Additionally, there is an observed widening of the productivity differential in favor of Zambia's major trading partners (Figure 5).

Figure 5: Impulse Response Functions



The effect of a contractionary monetary policy persists for two quarters and then declines progressively to die out after 20 quarters within the system. Its effect on inflation takes long as inflation remains subdued. Fiscal deficit increases in the first two quarters of monetary tightening contrary to expectation. This could be a reflection of poor monetary – fiscal coordination. Ideally, when monetary policy tightens, fiscal should implement complementary measures in order to help the economy deal with an adverse shock hitting the system. In the decade 2011 – 2021, this was largely not the case as fiscal deficit continued to mount amidst economic shocks (such as ending of the UMP by the Federal Reserve Bank in 2015) that may have required tightening of both monetary and fiscal policy. However, after two quarters, fiscal deficits responds to tight monetary policy with a decline probably on account of tight liquidity that may have led to some slow down on expenditure as financing the deficit becomes relatively expensive. Productivity reverts to its equilibrium within five quarters after the initial policy impulse whilst the REER reverts to the equilibrium within 15 quarters after appreciating.

The result of the effect of monetary policy shock on REER is relatively long lived, lasting 15 quarters. This result is in line with findings in the literature showing persistence of the real exchange rate in response to monetary policy shocks (Carvalho, Nechio and Yao, 2018). Thus, in Zambia, there is a role for monetary policy to offset uncompetitive price pressures.

VI. Conclusion

The real effective exchange rate (REER) is an important macroeconomic variable, pivotal in the overall resource allocation within an economy. It also serves as a measure of competitiveness, significantly shaping a country's external sector performance. This study sought to examine the determinants of equilibrium exchange rate, compute the exchange rate misalignment and assess the impact of monetary policy on the equilibrium REER in Zambian based on data over the period 2006-2021.

The study used fractional cointegration VAR (FCVAR) model because which accommodates both integer order and fractionally integrated data. The results show that the Zambian REER is fractionally cointegrated with the real copper price, fiscal deficit and productivity differential. The equilibrium relationship is stable but a shock to it tends to have a slow speed of adjustment to its steady state. Further, the study employed a Bayesian VAR with sign restrictions to identify the impact of monetary policy on the REER. The results show that a positive monetary impulse leads to the REER appreciation and convergence to the equilibrium occurs after 15 quarters.

However, our findings do not support the Balassa-Samuelson effect as an increase in the productivity differential in favor of Zambia seems to be associated with a depreciation of the REER. In Zambia, external factors relevant for the REER are mainly those arising from shocks to the real copper price. Anecdotally, shocks to the copper prices largely arise from exogenous shocks in the global economy.

Given that the equilibrium process of the REER reverts to the mean, albeit slowly, the major policy implication is that there is some scope for policy intervention to hasten the return to equilibrium when the economy is hit by shocks that result in non-competitive prices. These would include among others, foreign exchange interventions that averts excessive appreciation of the Kwacha-Dollar nominal exchange rate.

Monetary policy shocks have a significant effect on the REER, for instance, contractionary shocks result in an appreciation of the REER on impact, an indication of the dampening effect of policy on inflation. Thus, policy makers should be conscious of the fact that their policy actions may have a direct impact on the external sector competitiveness. For example, loose monetary policy may result in unwanted depreciation of the real effective exchange rate which may occasion external imbalances. In addition, monetary policy tightening leads to a rise in the productivity differential against Zambia . The policy implication with this outcome is that monetary authorities should make a trade-off between high inflation and loss of productivity for about a two year-period.

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Annexes

Annex 1: Beta Convergence Estimations

The Beta convergence estimation procedure serves to gauge the long run average (steady state value) toward which a time series may revert in the event of deviations caused by various shocks. Furthermore, it provides insight into the speed of adjustment to this long run steady state value. This information can offer clues regarding whether the series of interest exhibit persistence or follows a mean reverting process (see Marques, 2004; and Dias and Marques, 2010). In our context, the estimation of the long run steady state value and the speed of adjustment for the equilibrium REER is derived from equations below.

These equations (1 and 2) are as presented by Hlivnjak (2009), Monfort (2008) and Mbao (2021).

$$\Delta x_t = \alpha + \beta(x_{t-1}) + \theta_t \qquad (1)$$

where,

Δ is the first differences of the variable of interest;
α is constant term representing autonomous growth in the variable of interest;
β is speed of convergence/adjustment to long run mean; and
θ_t is the error term such that $\theta_t \sim N(0, \sigma^2)$.

Equation (1) can be expanded as

$$x_t - x_{t-1} = \alpha + \beta(x_{t-1}) + \theta_t$$

or

$$x_t = \alpha + x_{t-1} + \beta(x_{t-1}) + \theta_t$$

or

$$x_t = \alpha + (1 + \beta)x_{t-1} + \theta_t,$$

conditioned on

$$-1 \leq \beta \leq 0.$$

The interpretation of equation (1) is that the speed of adjustment is supposed to be negative if convergence to the long run occurs. The closer its absolute figure is to one (1) (i.e. 100%), the greater is the speed of adjustment, and the opposite is true. Specifically, the null hypothesis guiding the tests is that the equilibrium REER does not converge to its steady state value, meaning $\beta \geq 0$, against the alternative hypothesis that equilibrium REER converges to a long run steady state. This is the case for $-1 \leq \beta < 0$.

That is,

$$H_0: \beta \geq 0$$

$$H_a: -1 \leq \beta < 0$$

For estimating the long run steady state value for the sample, the equation below is used.

$$Long\ run\ steady\ state = -\frac{\alpha}{\beta}, \qquad (2)$$

where,

α and β are as defined before.

Fitting the model above with the equilibrium REER data and estimated with an OLS procedure in EViews, the results are presented in the table below.

Dependent Variable: D(EREER)
Method: Least Squares
Date: 09/25/22 Time: 17:53
Sample (adjusted): 2006Q2 2021Q4
Included observations: 63 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.465017	0.720928	3.419226	0.0011
EREER(-1)	-0.299635	0.087683	-3.417239	0.0011

R-squared	0.160676	Mean dependent var	0.001543
Adjusted R-squared	0.146916	S.D. dependent var	0.058602
S.E. of regression	0.054126	Akaike info criterion	-2.963777
Sum squared resid	0.178706	Schwarz criterion	-2.895741
Log likelihood	95.35898	Hannan-Quinn criter.	-2.937018
F-statistic	11.67752	Durbin-Watson stat	1.832802
Prob(F-statistic)	0.001132		

$$\alpha = 2.465017$$
$$\beta = -0.299635$$
$$-\frac{\alpha}{\beta} = -\left(\frac{2.465017}{-0.299635}\right) = 8.22673252$$

Annex 2: Literature Review on Fractional Integration

The issue of fractional integration and fractional cointegration being related to the subject of long memory process in the data presents a host of models/methods available for testing this phenomenon. A detailed review of such methods has been well documented by Baillie (1996) with the accompanied application of the methods to analyze economic and financial data.

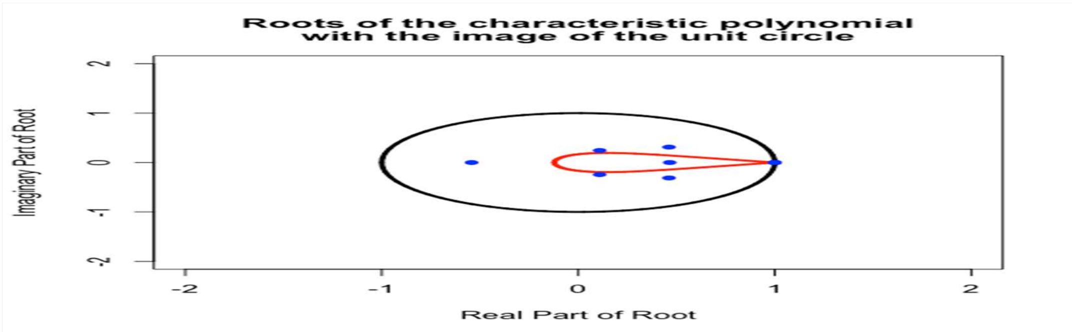
Early methods of determining fractional integration used fractal techniques for estimating the Hurst parameter, which is one of the measures of long memory in time series data. Since the pioneering work of the Rescaled Range (R/S) analysis by Hurst (1951) a lot more methods have been developed as pointed out by Cajueiro et al.(2009). Kirichenko, Radivilova and Deineko (2011) have all documented and assessed a range of techniques used for estimating the Hurst parameter with varying recommendations on their suitability based on whether the series under observation is stationary or otherwise. However, and more crucially, the use of the Hurst parameter to assess the fractional integration of the data should be done with a clear understanding of the true underlying behavior of the data under review.

Serinaldi (2010) and Eke et al. (2002) have guided on the need to establish whether the data is either fractional Brownian motion (*fBm*) or fractional Gaussian noise (*fGn*) when estimating long memory in the data based on the Hurst parameter. The *fGn* is the first difference of the *fBm*, which is a fractional differentiation of the Brownian motion or random walk Delignieres and Torre (2009). The *fBm* and the fGn have different statistical properties regarding stationarity or persistence and yet there can be the same Hurst parameter estimate of these two different data processes. This is the down side of the Hurst parameter estimate and therefore has the possibility of one making a spurious interpretation of the results if the signal classification of *fBm* and *fGn* properties are not established. Cannon et al. (1997) has empirically established that there is a unique Hurst exponent which characterises both the *fBm* and the *fGn* signals. In view of this, methods have been developed to estimate the fractal signal of time series data which can help to establish the nature of long memory and thus the type of fractional integration.

Data with long memory or fractionally integrated exhibit power law of the type $1/f^{\beta}$ (Chen et al., 1997; and Pilgram and Kaplan, 1998). According to Cannon et al.(1997), Eke et al.(2002) and Delignieres and Torre (2009) such processes can be either of the *fGn* or *fBm* classification such that $\beta \in (-1, 1)$ denotes the *fGn* series and $\beta \in (1, 3)$ implies the *fBm* type of signal (Serinaldi, 2010 and Cannon et al., 1997). White Gaussian noise is *fGn* type of the $1/f^{\beta}$ process with $\beta = 0$. The Brownian motion is the *fBm* process with $\beta = 2$ (Hausdorff *et al.*, 2000). Both the *fGn* series and the *fBm* signal with $\beta < 2$ are the anti-persistent processes. The *fBm* signals of $\beta > 2$ are regarded to be persistent over a relatively longer period. In this regard, estimates aimed at establishing whether time series data is of the *fBm* or *fGn* signal can be used to identify whether the series of interest is fractionally integrated since such methods help to identify series of long memory and classify them accordingly as described above.

Both the H and β parameters are not informative on the extent of fractional order of integration in a time series. However, autoregressive fractionally integrated moving average (ARFIMA) models of the type ARFIMA(p, d, q) have been estimated to determine the exact fractional order of integration for processes that are stationary. For example see Granger and Joyeux (1980) and Baillie (1996) for such estimations. The parameter p in the ARFIMA models is the number of autoregressive terms while d is the fractional difference operator and q is the number of lagged forecast errors in the prediction equation. Recently, models that considers fractional differencing parameter in a multivariate set up and also for non-stationary processes have been proposed by Johansen (2008), Johansen and Nielsen(2010) and Johansen and Nielsen(2018) and estimated by others as highlighted before in the earlier paragraphs. This approach informs the methodology in our current research.

Annex 3a: Characteristic Roots from the FCVAR Model



Annex 3b: Characteristic Roots from the CVAR Model